

Math 330 Quiz 3 Version A

1. Let

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 2 & 3 & -5 \\ -2 & 4 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}.$$

Solve the system $Ax = b$. Find x .

Handwritten solution for the system $Ax = b$:

Initial augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 0 & -2 & -3 \\ 2 & 3 & -5 & -3 \\ -2 & 4 & 0 & 7 \end{array} \right]$$

Row operations:

- $r_2 - r_1 \rightarrow r_2$
- $r_3 + r_1 \rightarrow r_3$

Intermediate augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 0 & -2 & -3 \\ 0 & 3 & -3 & 0 \\ 0 & 4 & -2 & 4 \end{array} \right]$$

Row operation:

- $r_3 - \frac{4}{3}r_2 \rightarrow r_3$

Final augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 0 & -2 & -3 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 2 & 4 \end{array} \right]$$

Back-substitution:

- $2x_3 = 4 \implies x_3 = 2$
- $3x_2 - 3x_3 = 0 \implies x_2 = 2$
- $2x_1 - 2x_3 = -3 \implies x_1 = \frac{1}{2}$

Solution:

$$x = \begin{bmatrix} \frac{1}{2} \\ 2 \\ 2 \end{bmatrix}$$

2. Let $B = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$. Find B^{-1} .

Handwritten solution for finding B^{-1} :

Initial augmented matrix:

$$\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array} \right]$$

Row operation:

- $r_2 - r_1 \rightarrow r_2$

Intermediate augmented matrix:

$$\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{array} \right]$$

Row operations:

- $\frac{1}{2}r_1 \rightarrow r_1$
- $\frac{1}{3}r_2 \rightarrow r_2$

Final augmented matrix:

$$\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{3} & \frac{1}{3} \end{array} \right]$$

Inverse matrix:

$$B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

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3. Let

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular U such that $LU = A$.

Handwritten solution on lined paper showing the LU decomposition of matrix A .

Initial matrix A :

$$\begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}$$

Row operation: $r_2 - 2r_1 \rightarrow r_2$

Elementary matrix E_1 and resulting matrix:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 4 & 1 & 8 \end{bmatrix}$$

Row operation: $r_3 - 4r_1 \rightarrow r_3$

Elementary matrix E_2 and resulting matrix:

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 5 & 16 \end{bmatrix}$$

Row operation: $r_3 - \frac{5}{3}r_2 \rightarrow r_3$

Elementary matrix E_3 and resulting matrix:

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{5}{3} & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$

The final upper triangular matrix U is:

$$U = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$

The lower triangular matrix L is the product of the inverses of the elementary matrices:

$$L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & \frac{5}{3} & 1 \end{bmatrix}$$