Math 330 Quiz 3 Version A

1. Let

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 2 & 3 & -5 \\ -2 & 4 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}.$$

Solve the system $Ax = b$. Find x.

$$\begin{bmatrix} 2 & 0 & -2 & -3 \\ 2 & 3 & -5 & -3 \\ -2 & 4 & 0 & 7 \end{bmatrix}$$

$r_2 - r_1 \to r_2$

$r_3 + r_1 \to r_3$

$$\begin{bmatrix} 2 & 0 & -2 & -3 \\ 0 & 3 & -3 & 0 \\ 0 & 4 & -2 & 4 \end{bmatrix}$$

$r_3 - \frac{4}{3} r_2 \to r_3$

$$\begin{bmatrix} 2 & 0 & -2 & -3 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$2x_3 = 4$; $x_3 = 2$

$3x_2 - 3x_3 = 0$; $x_2 = 2$

$2x_1 - 2x_3 = -3$; $x_1 = 1/2$

$$x = \begin{bmatrix} 1/2 \\ 2 \\ 2 \end{bmatrix}$$

2. Let $B = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$. Find B^{-1}.

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{bmatrix}$$

$r_2 - r_1 \to r_2$

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 3 & -1 & 1 \end{bmatrix}$$

$\frac{1}{2} r_2 \to r_2$

$\frac{1}{3} r_3 \to r_3$

$$\begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1/3 & 1/3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1/2 & 0 \\ -1/3 & 1/3 \end{bmatrix}$$

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3. Let

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular U such that $LU = A$.

$$\begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}$$

$r_2 - 2r_1 \to r_2$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 4 & 1 & 8 \end{bmatrix}$$

$r_3 - 4r_1 \to r_3$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 5 & 16 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$

$r_3 - \frac{5}{3} r_1 \to r_3$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5/3 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 6 \end{bmatrix} \qquad L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 5/3 & 1 \end{bmatrix}$$