Linear Algebra Formula Sheet

Orthogonal Projection

$$P = A(A^TA)^{-1}A^T$$

Least Squares

$$x = (A^TA)^{-1}A^Tb$$

Gram-Schmidt

$$\begin{aligned}
\tilde{q}_1 &= a_1 & q_1 &= \tilde{q}_1/\|\tilde{q}_1\| \\
\tilde{q}_2 &= a_2 - q_1(q_1 \cdot a_2) & q_2 &= \tilde{q}_2/\|\tilde{q}_2\| \\
\tilde{q}_3 &= a_3 - q_1(q_1 \cdot a_3) - q_2(q_2 \cdot a_3) & q_3 &= \tilde{q}_3/\|\tilde{q}_3\| \\
&\vdots & &\vdots \\
\tilde{q}_n &= a_n - q_1(q_1 \cdot a_n) - q_2(q_2 \cdot a_n) - \cdots - q_{n-1}(q_{n-1} \cdot a_n) & q_n &= \tilde{q}_n/\|\tilde{q}_n\|.
\end{aligned}$$

Least Squares using Gram-Schmidt

$$Rx = Q^Tb \qquad \text{where} \qquad R = Q^TA$$

Combinatorial Determinant Formula

$$\det A = \sum_{\text{all } n \times n \text{ permuation matrices } P} (\det P)(\text{product of the diagonal of } PA)$$

Cofactor (Laplace's) Expansion

$$\det A = \sum_{j=1}^{n} a_{ij}C_{ij} \qquad \text{where} \qquad C_{ij} = (-1)^{i+j}\det M_{ij}$$

and M_{ij} is the submatrix of A resulting from deleting row i and column j.

Formula for the Inverse

$$A^{-1} = \frac{C^T}{\det(A)}$$

In class we defined C_{ij} with the meaning of the indices reversed. Thus, i corresponded to the deleted column and j to the deleted row in the minor. In this case, the transpose isn't necessary in the above formula. This also changes the cofactor expansion.

Cramer's Rule

$$x_1 = \frac{\det B_1}{\det A} \qquad x_2 = \frac{\det B_2}{\det A} \qquad . \quad . \quad . \qquad x_n = \frac{\det B_n}{\det A}$$