Math 330 Exam 2 Version B

1. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 2 & 2 & 2 & 2 & 3 \\ 3 & 3 & 2 & 0 & 6 \end{bmatrix}$$

Find the reduced row echelon form R of A.

2. Let

$$A = \begin{bmatrix} 1 & 1 & -2 & 1 & 0 \\ 1 & 2 & 1 & 2 & 3 \\ 3 & 3 & 2 & -5 & 6 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} -11 \\ 7 \\ 53 \end{bmatrix}$$

The nullspace of A is

$$\mathcal{N}(A) = \left\{ \begin{bmatrix} 1 & 4 \\ 1 & -5 \\ 1 & 0 \\ 0 & 1 \\ -4/3 & 4/3 \end{bmatrix} v : v \in \mathbf{R}^2 \right\}.$$

One solution to $Ax = b$ is

$$x = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \end{bmatrix}.$$

Find all solutions.

3. Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 2 & 3 \\ 0 & 0 & 2 & 2 & 0 & 1 \end{bmatrix}.$$

The reduced row echelon form of A is

$$R = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

(i) Find the rank of A.

(ii) Find a basis for the nullspace $\mathcal{N}(A)$. What is the dimension $\dim \mathcal{N}(A)$?

(iii) Find a basis for the column space $\mathcal{C}(A)$. What is the dimension $\dim \mathcal{C}(A)$?

4. Answer the following questions about linear independence, span and basis. Give a reason each of your answers.

(i) Do the vectors

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

span the space R^4?

(ii) Do the vectors $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}0\\1\\1\end{bmatrix}$ span the same space as $\begin{bmatrix}2\\0\\1\end{bmatrix}$ and $\begin{bmatrix}0\\5\\1\end{bmatrix}$?

(iii) Are the vectors

$$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}$$

linearly independent?

5. True or false with a counterexample if false and a reason if true.

(i) If A is any matrix, then $\mathcal{N}(A) = \mathcal{N}(A^T)$.

(ii) If A is any $n \times m$ matrix, then $\dim \mathcal{N}(A) = \dim \mathcal{N}(A^T)$.

(iii) If P is a $n \times n$ permutation matrix, then $P^n = I$.

(iv) If A is an $n \times n$ invertible matrix, then $\text{rank}(A) = n$.

6. Let

$$A = \begin{bmatrix}1 & 2 & 3\\1 & 6 & 8\\1 & 6 & 2\end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular U such that $LU = A$.

7. Let $B = \begin{bmatrix}1 & 1 & -2\\1 & 2 & 1\\3 & 3 & 2\end{bmatrix}$. Find B^{-1}.

8. Let $A = LU$ where

$$L = \begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\3 & -1 & 1\end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix}2 & 0 & 1\\0 & 2 & \sqrt{2}\\0 & 0 & 3\end{bmatrix}.$$

Solve the system

$$Ax = \begin{bmatrix}2\\0\\0\end{bmatrix}.$$

Find x.