Math 330 Homework 5 Version A

1. Let $A, B, C \in M_{3\times 3}(\mathbf{R})$ such that $\det A = 2$, $\det B = 4$, $\det C = 5$. Find

(i) $\det(AB)$

(ii) $\det(2B)$

(iii) $\det(A^t)$

(iv) $\det(A^tC)$

(v) $\det(C^{-1})$

2. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & -1 & 3 \\ 1 & -1 & -1 \end{bmatrix}$. Express A as a product of elementry row matrices.

3. Find the following determinants. Do the calculations by hand; not with a computer.

(i) $\begin{vmatrix} 2 & 0 & 3 & 4 \\ 0 & 3 & -1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & \sqrt{7} \end{vmatrix}$

(ii) $\begin{vmatrix} 2 & 1 & -2 & 6 \\ -1 & 0 & -1 & -2 \\ 2 & 0 & 2 & 7 \\ 1 & 1 & 2 & 3 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$

4. Let $A \in M_{m\times n}(\mathbf{R})$ and $P \in M_{m\times m}(\mathbf{R})$.

(i) If P is non-singular prove that $\mathcal{N}(A) = \mathcal{N}(PA)$.

(ii) Give an example with P singular such that $\mathcal{N}(A) \neq \mathcal{N}(PA)$.

5. Suppose $A \in M_{3\times 3}(\mathbf{R})$ such that $A^t A = I$.

(i) Prove that $A^t(A - I) = -(A - I)^t$.

(ii) Prove that $\det A = \pm 1$.

(iii) Use **(i)** to prove that if $\det A = 1$ then $\det(A - I) = 0$.

(iv) Extra Credit: If $A \in M_{n\times n}(\mathbf{R})$ where $n > 3$ and $A^t A = I$ can you still prove parts **(i)**–**(iii)** above? If so, provide the proofs; if not, give a counter example.

6. Compute the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 1 & 2 & -1 \\ 4 & 7 & -2 \end{bmatrix}$$

by first computing the adjoint matrix.

7. Extra Credit: Work problem 11 from Mathews page 69.