Math 330 Quiz 3 Version A

1. The orthogonal complement $S^\perp$ of a subspace $S \subset \mathbf{R}^m$ is defined

(A) $S^\perp = \{\, v \cdot y : v \in \mathbf{R}^m \text{ and } y \in S \,\}$.

(B) $S^\perp = \{\, y \in \mathbf{R}^m : v \cdot y = 0 \text{ for all } v \in S \,\}$.

(C) $S^\perp = \{\, y \in \mathbf{R}^m : v \cdot y = 0 \text{ for at least one } y \in S \,\}$.

(D) both (B) and (C).

(E) none of these.

2. Let $A \in \mathbf{R}^{m\times n}$. Show that $\mathcal{C}(A)^\perp = \mathcal{N}(A^T)$.

3. Suppose $A \in \mathbf{R}^{m\times n}$ is factored as $A = QR$ where $Q \in \mathbf{R}^{m\times n}$ is has orthonormal columns and $R \in \mathbf{R}^{n\times n}$ is upper triangular. If

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}, \qquad R = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

then find the point $x \in \mathbf{R}^n$ that minimizes the norm $\|Ax - b\|$.

4. Find an orthonormal basis for the space spanned by the vectors

$$\left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right\}.$$

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5. Let $A \in \mathbf{R}^{m\times n}$ where $m \neq n$. Let

$$v \in \mathcal{N}(A^T), \quad w \in \mathcal{C}(A^T), \quad x \in \mathcal{N}(A) \quad \text{and} \quad y \in \mathcal{C}(A).$$

(i) How many components does the vector v have?

(ii) How many components does the vector w have?

(iii) How many components does the vector x have?

(iv) Home many components does the vector$y$ have?

(v) What is $v \cdot y$?

(vi) What is Ax?

(vii) Given any vector $z \in \mathbf{R}^n$ is it true that $z = v + y$ for some $v \in \mathcal{N}(A^T)$ and $y \in \mathcal{C}(A)$? Explain your answer.