Math 330 Homework 3

1. Circle the correct answer for the following multiple choice questions.

(i) Find the orthogonal complement of $\mathcal{C}(A)$.

(A) $\mathcal{N}(A)$
(B) $\mathcal{C}(A)$
(C) $\mathcal{N}(A^T)$
(D) $\mathcal{C}(A^T)$
(E) none of these

(ii) Find the orthogonal complement of $\mathcal{C}(A^T)$.

(A) $\mathcal{N}(A)$
(B) $\mathcal{C}(A)$
(C) $\mathcal{N}(A^T)$
(D) $\mathcal{C}(A^T)$
(E) none of these

(iii) Find Q^TQ where Q is an orthogonal matrix.

(A) I
(B) 0
(C) A
(D) Q^{-1}
(E) none of these

(iv) Find an example of non-singular matrix $A \in \mathbf{R}^{n\times n}$ such that $A^TA \neq AA^T$.

2. Let

$$a = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Project the vector b onto the subspace given by the span of a.

3. Let Q be the matrix with orthonormal columns given by

$$Q = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 2\sqrt{2} & 1 \\ 2 & -\sqrt{2} & 2 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Find the x which minimizes $\|Qx - b\|$.

4. Find an orthonormal basis for the space spanned by the vectors

$$\begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

5. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 7 & 8 & 9 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Find $\det A$.

6. Let

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 4 & 2 \end{bmatrix}$$

Find $\det B$.

7. Let A and B be 4×4 matrices. Suppose $\det A = 2$ and $\det B = 3$.

(i) Find $\det(-A)$.

(ii) Find $\det(A^3 B)$.

(iii) Find $\det(B^T)$.

(iv) Find $\det(B^{-1})$.

(v) Find $\det(A + A)$.

8. Let

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Let B_j be the matrices formed according to Cramer's rule with the j-th column of A replaced by b. If

$$\det B_1 = 18, \qquad \det B_2 = -18, \qquad \text{and} \qquad \det B_3 = 36$$

then what is b? Find b_1, b_2 and b_3.