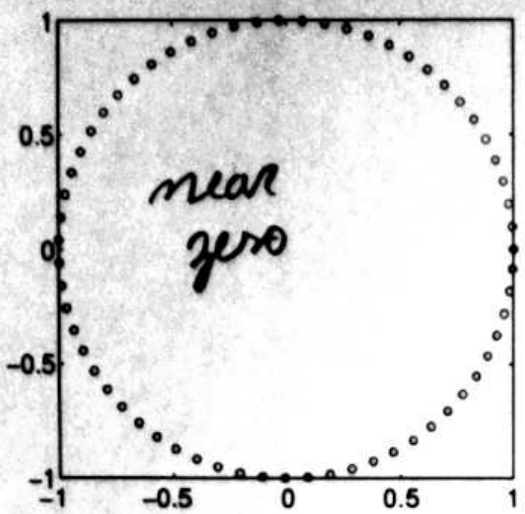
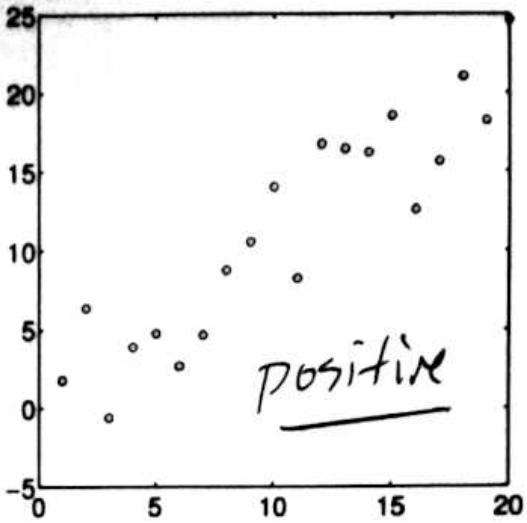
Math 330 Linear Algebra Homework 5

3.1 *Correlation coefficient.* Each of the following plots shows the points corresponding to two vectors x and y of the same size, *i.e.*, we plot points at the locations (x_i, y_i). In each case, determine whether the correlation coefficient ρ of the two vectors is positive (and, say, ≥ 0.5), negative (say, ≤ -0.5), or near zero (say, less than 0.3 in absolute value). (You must choose one of these options.)

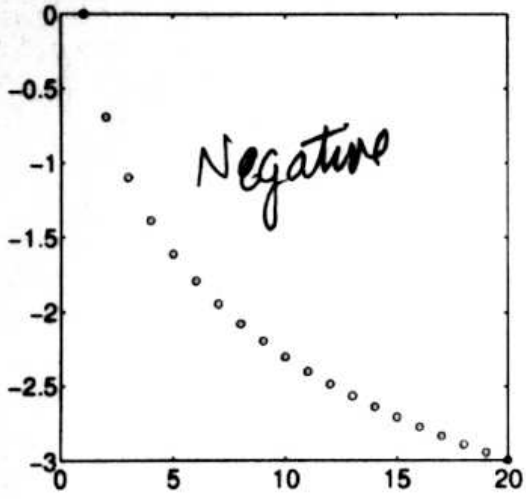
(a)



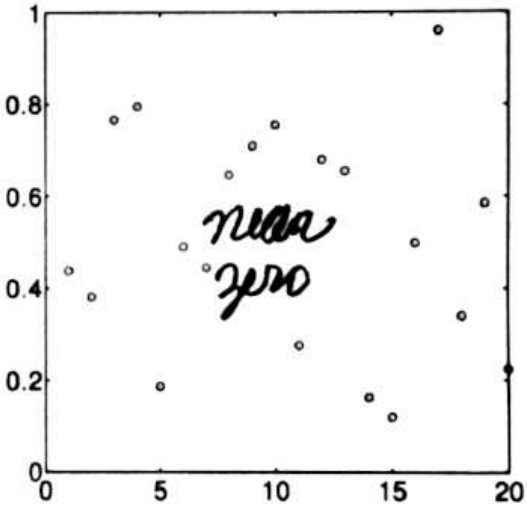
(b)



(c)



(d)



3.2 *Nearest neighbor and smallest angle*. Using Julia, find the nearest neighbor of $a = (1, 3, 4)$ among the vectors

$$x_1 = (4, 3, 5), \quad x_2 = (0.4, 10, 50), \quad x_3 = (1, 4, 10), \quad x_4 = (30, 40, 50).$$

Report the minimum distance of a to $x_1, \ldots, x_4$. Also find which of $x_1, \ldots, x_4$ makes the smallest angle with a and report that angle.

In [1]:
```
a=[1,3,4];
x1=[4,3,5];
x2=[0.4,10,50];
x3=[1,4,10];
x4=[30,40,50];
```

In [2]:
```
using LinearAlgebra
```

Compute the distance from a to each of the points x_i as

In [3]:
```
norm(a-x1)
```

Out[3]: 3.1622776601683795

In [4]:
```
norm(a-x2)
```

Out[4]: 46.53342884421908

In [5]:
```
norm(a-x3)
```

Out[5]: 6.082762530298219

In [6]:
```
norm(a-x4)
```

Out[6]: 65.7723346096214

The smallest is the distance between a and x_1. Therefore,

$$\min\{\|a - x_i\| : i = 1, \ldots, 4\} = \|a - x_1\| \approx 3.1622776601683795.$$

We now compute the angles θ_i using the formula

$$a^T x_i = \|a\| \|x_i\| \cos\theta_i$$

or equivalently $\theta = g(x_i)$ where

$$g(x) = \arccos\left(\frac{a^T x}{\|a\|\|x\|}\right).$$

In [11]:
```
g(x)=acos(a'*x/(norm(a)*norm(x)));
```

In [12]:
```
g(x1)
```

Out[12]: 0.4146551062003066

In [13]:
```
g(x2)
```

Out[13]: 0.4818770434517026

In [14]:
```
g(x3)
```

Out[14]: 0.2804174688623895

In [15]:
```
g(x4)
```

Out[15]: 0.24256387409548533

The value for i such that $g(x_i)$ is smallest is $i = 4$. Therefore, x_4 makes the smallest angle with a.

In [ ]:
```

```