

Math 330 Linear Alg.

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MWF 1:00-1:50PM in DMS 238 and through Zoom by appointment

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Upon completion of this course, students will be able to

- Solve linear systems using Gaussian elimination.
- Minimize least squares by Gram-Schmidt orthogonalization.
- Find LU, QR and  $U\Sigma V^T$  matrix factorizations.

Note that Gaussian elimination provides LU factorization

Gram-Schmidt orthogonalization leads to QR factorization

That other matrix factorization is related to eigenvalues and eigenvectors.

Important: There are three web pages related to this course.

- 1 The WebCampus webpage will contain the links to turn in the scanned written homework, the Zoom meeting links and used to communicate via electronic messages.
- 2 There is a link to <https://fractal.math.unr.edu/~ejolson/330-23/> from the homepage on WebCampus which is where I will post my announcements about exams, homework assignments and lecture notes (like this one).
- 3 The textbook is available from the Pearson MyLab website. There will also be online homework that needs to be done there.

We will follow the textbook starting from Chapter 1 and ending in Chapter 7. Some sections will be skipped--especially applications that you'll likely see in the engineering, science and economics classes for which this course is required. Some topics introduced in early chapters will be returned to in subsequent chapters and elaborated on.

Teaching a little bit of one topic, then doing something else, and then returning to the first topic again is called the spiral method of teaching mathematics. Although it's like going in circles, this allows each topic time to sink in before it is further developed.

We will do our best to follow the order of presentation in the text.

Let's look at the book and see what's coming up:

## Chapter 1 Linear Equations in Linear Algebra 1

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### INTRODUCTORY EXAMPLE: Linear Models in Economics and Engineering 1

- 1.1 Systems of Linear Equations 2
- 1.2 Row Reduction and Echelon Forms 13
- 1.3 Vector Equations 26
- 1.4 The Matrix Equation  $Ax = b$  37
- 1.5 Solution Sets of Linear Systems 45
- 1.6 Applications of Linear Systems 53
- 1.7 Linear Independence 60
- 1.8 Introduction to Linear Transformations 67
- 1.9 The Matrix of a Linear Transformation 75
- 1.10 Linear Models in Business, Science, and Engineering 85
- Projects 93
- Supplementary Exercises 93

## Chapter 2 Matrix Algebra 97

INTRODUCTORY EXAMPLE: Computer Models in Aircraft Design 97

- 2.1 Matrix Operations 98
- 2.2 The Inverse of a Matrix 111 (Continuation of 1.5.)
- 2.3 Characterizations of Invertible Matrices 121
- 2.4 Partitioned Matrices 126
- \* → 2.5 Matrix Factorizations 132
- 2.6 The Leontief Input–Output Model 141
- 2.7 Applications to Computer Graphics 147

- ↳ { 2.8 Subspaces of  $\mathbb{R}^n$  155 (Continuation of lin. independence)
- 2.9 Dimension and Rank 162
- Projects 169
- Supplementary Exercises 169

Note how sections from chapter 1 are revisited in chapter 2.

## Chapter 3 Determinants 171

INTRODUCTORY EXAMPLE: Weighing Diamonds 171

short, because determinants are mostly used for theory...

- 3.1 Introduction to Determinants 172
- 3.2 Properties of Determinants 179
- 3.3 Cramer's Rule, Volume, and Linear Transformations 188
- Projects 197
- Supplementary Exercises 197

This chapter could be much longer, but we do only what is necessary to develop techniques for finding eigenvalues later in chapter 5 later.

## Chapter 4 Vector Spaces 201

INTRODUCTORY EXAMPLE: Discrete-Time Signals and Digital Signal Processing 201

|   |     |  |     |             |
|---|-----|--|-----|-------------|
| → | 4.1 | Vector Spaces and Subspaces  | 202 |             |
|   | 4.2 | Null Spaces, Column Spaces, Row Spaces, and Linear Transformations | 211 |             |
|   | 4.3 | Linearly Independent Sets; Bases                                   | 222 | (again 1.7) |
|   | 4.4 | Coordinate Systems   | 231 |             |
|   | 4.5 | The Dimension of a Vector Space                                    | 241 |             |
|   | 4.6 | Change of Basis  | 249 |             |
|   | 4.7 | Digital Signal Processing  | 255 |             |
|   | 4.8 | Applications to Difference Equations                               | 262 |             |
|   |     | Projects   | 271 |             |
|   |     | Supplementary Exercises  | 271 |             |

Most of Chapter 4 is an elaboration on what was already introduced in Chapters 1 and 2.

## Chapter 5 Eigenvalues and Eigenvectors 273

INTRODUCTORY EXAMPLE: Dynamical Systems and Spotted Owls 273

|   |     |   |     |            |
|---|-----|---|-----|------------|
| ~ | 5.1 | Eigenvectors and Eigenvalues            | 274 |            |
| - | 5.2 | The Characteristic Equation             | 282 |            |
| - | 5.3 | Diagonalization                         | 290 |            |
| - | 5.4 | Eigenvectors and Linear Transformations | 297 |            |
| ? | 5.5 | Complex Eigenvalues                     | 304 |            |
|   | 5.6 | Discrete Dynamical Systems              | 311 |            |
|   | 5.7 | Applications to Differential Equations  | 321 | Skip these |
|   | 5.8 | Iterative Estimates for Eigenvalues     | 329 |            |
|   | 5.9 | Applications to Markov Chains           | 335 |            |
|   |     | Projects                                | 345 |            |
|   |     | Supplementary Exercises                 | 345 |            |

We don't have time for many applications, but if there is one you particularly want to learn about, please send me a note on WebCampus saying so. I'll try to add it to the course.

## Chapter 6 Orthogonality and Least Squares 349

**INTRODUCTORY EXAMPLE:** Artificial Intelligence and Machine Learning 349

|          |  |     |
|----------|--|-----|
| 6.1      | Inner Product, Length, and Orthogonality | 350 |
| 6.2      | Orthogonal Sets                          | 358 |
| 6.3      | Orthogonal Projections                   | 367 |
| ✖✖ → 6.4 | The Gram–Schmidt Process                 | 376 |
| ✖✖ → 6.5 | Least-Squares Problems                   | 382 |
| 6.6      | Machine Learning and Linear Models       | 390 |
| 6.7      | Inner Product Spaces                     | 399 |
| 6.8      | Applications of Inner Product Spaces     | 407 |
|          | Projects                                 | 413 |
|          | Supplementary Exercises                  | 414 |

This section was the first chapter in the linear algebra book by Boyd and Vandenberghe. It focuses on the second learning objective--optimization, Gram-Schmidt and the QR factorization.

Note that the first learning objective--Gaussian Elimination--was already covered in sections 1.5, 2.2 and 2.5.

## Chapter 7 Symmetric Matrices and Quadratic Forms 417

**INTRODUCTORY EXAMPLE:** Multichannel Image Processing 417

|           |   |     |
|-----------|---|-----|
| → 7.1     | Diagonalization of Symmetric Matrices           | 419 |
| 7.2       | Quadratic Forms                                 | 425 |
| 7.3       | Constrained Optimization                        | 432 |
| END → 7.4 | The Singular Value Decomposition                | 439 |
| 7.5       | Applications to Image Processing and Statistics | 449 |
|           | Projects  | 457 |
|           | Supplementary Exercises                         | 457 |

We won't cover very much from Chapter 7 but just enough to do the singular value decomposition. This is the last matrix factorization and surprisingly important in applications involving digital filtering and compression. For example, this factorization is related to the proper orthogonal decomposition which is used in machine learning and data mining.

On Wednesday we will start chapter 1.

We will not spend too much time of chapter 1, because those topics will be reviewed again in chapter 2 and 4.