Solve $Ax=b$:

$$A=\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix} \qquad b=\begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$

algebraic form of $Ax=b$

$$\begin{cases} x_1 + 2x_2 + 4x_3 = -2 \\ \quad\;\; x_2 + 5x_3 = 2 \\ -2x_1 - 4x_2 - 3x_3 = 9 \end{cases}$$

help to line up the variables sometime the printing on an exam or in the book doesn't do this

① Augmented Matrix

pivot

$$[A|b] = \left[\begin{array}{ccc|c} \boxed{1} & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right]$$

eliminate the 2 in the pivot column ...

$r_3 \leftarrow r_3 + 2r_1$

② Make echelon form:

leading entries

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

consistent since no zero row on the left is paired with something nonzero on the right.

Is this in echelon form?

These zero's were lucky they appeared without additional work

1. All nonzero rows are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it. makes a triangle shape in lower left.
3. All entries in a column below a leading entry are zeros.

③ Reduced Echelon form:

make these entries zero using elimination

but first rescale last row.

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{array}\right] \qquad r_3 \leftarrow \tfrac{1}{5} r_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{array}{l} r_2 \leftarrow r_2 - 5r_3 \\ r_1 \leftarrow r_1 - 4r_3 \end{array}$$

eliminate here ...

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad r_1 \leftarrow r_1 - 2r_2$$

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad -6 - 2(-3) = 0$$

④ Algebraic form

$x_1 = 0$

$x_2 = -3$

$x_3 = 1$

⑤ Solution to $Ax = b$:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$$

... done

Another example: $Ax = b$

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{bmatrix} \qquad b = \begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$$

① Augmented matrix

$$[A \mid b] = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

$r_2 \leftarrow r_2 - 3r_1$

$r_3 \leftarrow r_3 - 5r_1$

② Echelon form

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix}$$

$r_2 \leftarrow -\frac{1}{4} r_2$

$r_3 \leftarrow -1 r_3$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 8 & 16 & 34 \end{bmatrix}$$

$r_3 \leftarrow r_3 - 8 r_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 10 \end{array}\right]$$

is this consistent

this row has zeros on the left and non-zero on the right..

Inconsistent ... no solution...

If A is an $m \times n$ matrix, $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^n$, and c is a scalar, then:

a. $A(\mathbf{u}+\mathbf{v}) = A\mathbf{u} + A\mathbf{v}$; $\quad f(u+v) = f(u) + f(v)$

b. $A(c\mathbf{u}) = c(A\mathbf{u})$. $\quad f(cu) = cf(u)$

$f(x) = Ax$

The definition of a linear function.

... already had it

Next time Section 1.5...