Some how my note-taking program crashed when the emergency fire message started and I lost the first part of the lecture notes. I'll try to recreate them here as best I remember.

We started by finishing up chapter 2. There was one example that needed to be finished by computing the nullspace matrix N.

$$32. A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We recalled why the basis for the column space is made from the columns of A corresponding to the pivot variables. The point is that the relationships between the columns of a matrix are unaffected by the row operations and that it's clear the pivot columns in the echelon and reduced echelon forms are independent and that the columns corresponding to the free variables can be written interms of the columns with the pivots.

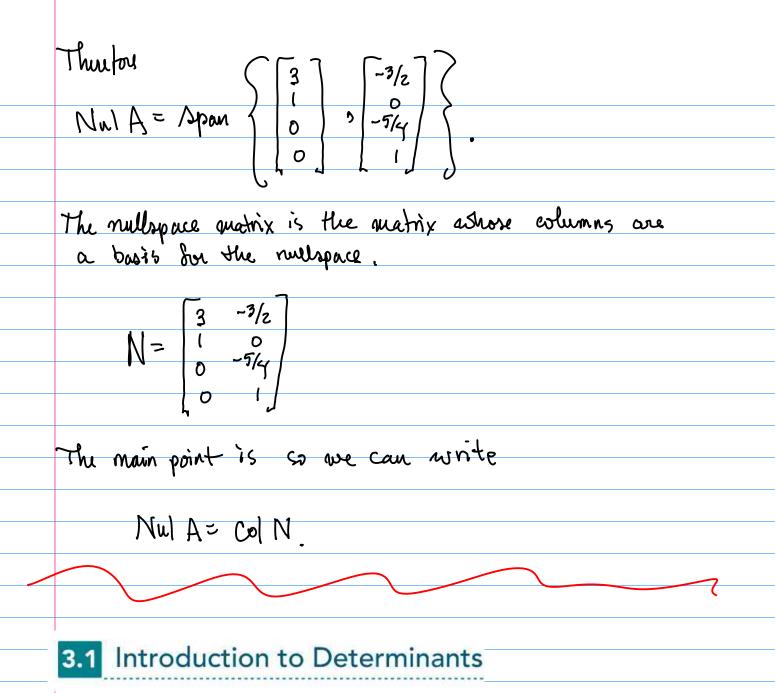
Thus

$$ColA = Apan \left\{ \begin{bmatrix} -3\\2\\3 \end{bmatrix}, \begin{bmatrix} -2\\4\\-2 \end{bmatrix} \right\}$$

and $dim colA = VankA = 2 = # of pirot variables$

Now ove do the nullspace. First
dim Nail A = # of free variables
= 1 - # of piror variables = 2.
By definition Nail A is the solutions to the homogeneous problem Az=0.
Nul A =
$$\begin{bmatrix} x : Ax = 0 \\ x : Ax = 0 \end{bmatrix}$$

reduced celebro form
We already know
 $\begin{bmatrix} 3 & 9 & 2 \\ 2 & -6 \\ 3 & -9 & 2 \end{bmatrix}$
 $\begin{bmatrix} -3 & 0 & 3/2 \\ 1 & -3 & 0 & 3/2 \\ 0 & 0 & 1 & 5/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
In algebraic form we have
 $x_1 - 3x_2 + 3/2 x_4 = 0$
 $x_3 + 5/4x_4 = 0$
 $x_3 + 5/4x_4$
The solution to $Ax = 0$ is
 $x_1 = 3x_2 - 3/2 x_4$
 $x_3 = -5/4 x_4$
 $x_4 = -5/4 x_4$
 $x_5 = -5/4 x_4$
 $x_7 = \begin{bmatrix} x_7 \\ x_8 \\ x_9 \end{bmatrix} = \begin{bmatrix} 3x_2 - 3/2 x_4 \\ -5/4 x_4 \\ -5/4 x_4 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3/2 \\ 0 \\ -5/4 \end{bmatrix}$



The chapter is very condensed because determinants are less useful for practical computations than for theory.

There is another linear algebra course on theory called Math 430. That course will build more intuition about determinants but also focuses on other things.

Somehow determinants always get short changed.

Consider a 2x2 meetrix A= [a b] and solving Az=b we're not interested right now about the night-hand side of the equation so write the augmented matrix as $\left[A(b) \ge \begin{vmatrix} a & b \\ c & d \end{vmatrix}^{2}\right]$ Now, if a #0 we could perform the elimination step c d rzerz - ar $\begin{bmatrix} a & b \\ 0 & d - \frac{c}{b} \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & ad - bc \end{bmatrix}$ We arrive at an augmented queitix that looks like a b (?) 0 ad-bc (n) which means $ax_1 + bx_2 = ?$ ad-bc x2 = ?

Solving for scr yields

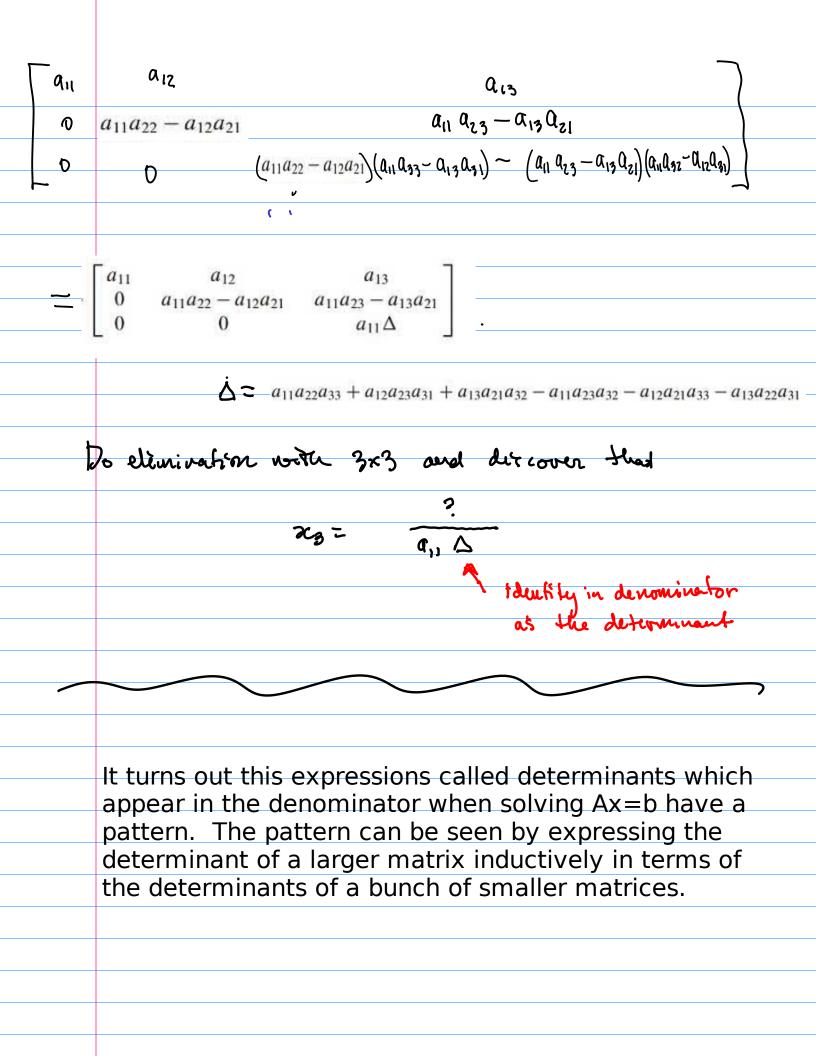
$$\chi_{2} = \frac{\alpha_{1}^{2}}{\alpha_{2} - bc}$$
This in the denominator is
called the determinant...
We like to call our friends by name
and this denominator is so friendly
it keeps appearing...
What happens it we swap rows first?

$$\begin{bmatrix} \alpha & b \\ c & d \end{bmatrix} \qquad r_{1} \leftarrow r_{2}$$
Now eltminate...

$$\begin{bmatrix} c & d \\ 0 & b - \frac{2}{c}d \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & bc-ad \end{bmatrix}$$

$$x_{1} \qquad x_{2}$$
Now solving for xs yields

$$\chi_{2} = \frac{c^{2}}{bc-ad} = \frac{-c^{2}}{ad-bc}$$
have is the deferminant again.



$$A = \begin{bmatrix} 1 & 2 & 6 \\ 5 & 4 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A_{2,3} = \begin{bmatrix} 1 & 2 & 5 \\ 5 & 4 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 5}$$

$$A_{3,1} = \begin{bmatrix} 2 & 6 \\ 5 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & 1 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$$