

Suppose the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some given \mathbf{b} , and let \mathbf{p} be a solution. Then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where \mathbf{v}_h is any solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

5.
$$x_1 + 3x_2 + x_3 = 0$$

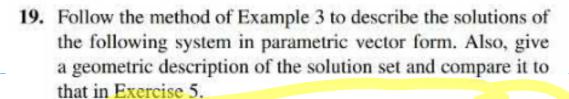
$$-4x_1 - 9x_2 + 2x_3 = 0$$

$$-3x_2 - 6x_3 = 0$$
Solve $Ax = 0$
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Solve $Ax = 0$

I already know x=0 15 one solution. One there any others?

Solve Az= 0.

Make the echelon/reduced echelon form 0 -3 -6:0 ra < 12 + 41, 0 3 6 0 0 -5 : 0 $r_2 \leftarrow \frac{1}{3} r_2$ of the free variables 50 luhion x1= 5 x3 -> $\mathcal{X} = \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix} = \begin{bmatrix} 5\chi_3 \\ -2\chi_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -\lambda \\ \chi_3 \end{bmatrix}$ Nul A= {x ∈ Rn : Ax = on } $= \begin{cases} 5 \\ -2 \end{cases} \chi_3 : \chi_3 \in \mathbb{R} \end{cases} = \begin{cases} 5 \\ -2 \end{cases} \left[\chi_3 \right] : \chi_3 \in \mathbb{R} \end{cases} = \operatorname{Col} \left[\begin{bmatrix} 5 \\ -2 \end{bmatrix} \right]$



$$x_1 + 3x_2 + x_3 = 1 x_4$$

$$-4x_1 - 9x_2 + 2x_3 = -1 x_4$$

$$-3x_2 - 6x_3 = -3 x_4$$

$$x_4 = 1$$

5.
$$x_1 + 3x_2 + x_3 = 0$$

 $-4x_1 - 9x_2 + 2x_3 = 0$
 $-3x_2 - 6x_3 = 0$

Make the echelon/reduced echelon form

$$r_3 \leftarrow r_3 + r_2$$

$$r_1 \leftarrow r_1 - r_2$$

$$r_2 \leftarrow \frac{1}{3} r_2$$

Lucky, otherwise there'd be no soludions

$$x_1 - 5x_3 = -2$$
 $x_2 + 2x_3 = 1$

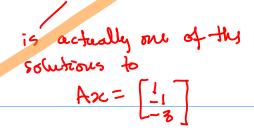
solve pivot variables in terms

$$x_1 = -\lambda + 5003$$

$$x_2 = 1 - \lambda x_3$$

$$\chi = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2+5x_3 \\ 1-2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} \times_3$$

Nul A



Suppose the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some given \mathbf{b} , and let \mathbf{p} be a solution. Then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where \mathbf{v}_h is any solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

So the theorem explains what happens when we solve the problem as in the previous section. The idea is to explain a general understanding of what's going on. That is what theory means (in mathematical contexts).