converges for $x \in (-1, 1)$.

$$\ln t = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (t-1)^n \qquad t = 1+x \qquad x = t-1$$

converges for $t \in (0,2)$

Therefore if $\lambda_i \in (0,2)$ then

$$\ln A = P \begin{bmatrix} \ln\lambda_1 & & & \\ & \ln\lambda_2 & & \\ & & \ddots & \\ & & & \ln\lambda_n \end{bmatrix} P^{-1} \quad \text{is okay}$$

Suppose $\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$ here $\lambda_3, \lambda_2 \notin (0,2)$

$$\ln t = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (t-1)^n \quad \text{converges for } t \in (0,2)$$

$$\ln \frac{t}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left(\frac{t}{4}-1\right)^n \quad \text{converges for } \frac{t}{4} \in (0,2)$$

Thus

$$\ln t = \ln 4 + \ln \frac{t}{4} = \ln 4 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left(\frac{t}{4}-1\right)^n$$

converges for $t \in (0,8)$

Now $\lambda_1, \lambda_2, \lambda_3 \in (0,8)$ so

$$\ln A = P \begin{bmatrix} \ln\lambda_1 & 0 & 0 \\ 0 & \ln\lambda_2 & 0 \\ 0 & 0 & \ln\lambda_3 \end{bmatrix} P^{-1}$$

Conclusion if $\lambda_i > 0$ for $i = 1, \dots, n$ then

$$\ln A = P \begin{bmatrix} \ln \lambda_1 & & \bigcirc \\ & \ln \lambda_2 & \\ \bigcirc & & \ddots \\ & & & \ln \lambda_n \end{bmatrix} P^{-1}$$

If λ_i are complex or negative defining $\ln A$ is more difficult as is $\ln \lambda_i$.

problem # 3 on page 310 section 5.5 3. $\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 5 \\ -2 & 3-\lambda \end{bmatrix} = (1-\lambda)(3-\lambda) - (-2)(5)$$

$$= \lambda^2 - 4\lambda + 3 + 10 = \lambda^2 - 4\lambda + 13 = 0$$

$a = 1$ $b = -4$ $c = 13$

Quadratic formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{4 \cdot 4 - 4 \cdot 13}}{2}$$

$$= \frac{4 \pm \sqrt{4 \cdot 4 - 4 \cdot 13}}{2} = \frac{4 \pm 2\sqrt{4 - 13}}{2} = 2 \pm \sqrt{-9}$$

$$= 2 \pm 3i \quad \text{where } i^2 = -1.$$

Find the eigenvectors

$\lambda_1 = 2-3i \qquad \text{Nul}(A-\lambda I) = \text{Nul}\left(\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} - (2-3i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$$\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} - (2-3i)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1+3i & 5 \\ -2 & 1+3i \end{bmatrix}$$

Check that $\det(A-\lambda I)=0$

$$\det\begin{bmatrix} -1+3i & 5 \\ -2 & 1+3i \end{bmatrix} = (-1+3i)(1+3i)+10$$

$$= -1+9i^2 \cancel{-3i} + \cancel{3i} + 10 \quad = -1-9+10 = 0$$

Solve the homogeneous equation $(A-\lambda I)x=0$ to find x

$$-2x_1 + (1+3i)x_2 = 0$$

$$x_1 = \frac{1+3i}{2}x_2$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1+3i}{2}x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1+3i}{2} \\ 1 \end{bmatrix} x_2$$

eigenvector

eigenvalue $\lambda_1 = 2-3i$ rescaled eigenvector $v_1 = \begin{bmatrix} 1+3i \\ 2 \end{bmatrix}$

$(-1+3i)x_1 + 5x_2 = 0$ another eigenvector $\begin{bmatrix} 5 \\ 1-3i \end{bmatrix}$

these are multiples of each other by a complex multiple.

How?

$$\frac{1}{1-3i} = \frac{1}{1-3i}\cdot\frac{1+3i}{1+3i} = \frac{1+3i}{1+3^2} = \frac{1+3i}{10} = \frac{1}{10} + \frac{3}{10}i$$

Square could be large

these numbers are not large

$\lambda_1 = 2-3i$
$\lambda_2 = 2+3i$ so $\lambda_2 = \overline{\lambda_1}$ ← complex conjugate made by switching i for $-i$ everywhere

$$A v_1 = \lambda_1 v_1 \qquad \lambda_1 = 2-3i \quad v_1 = \begin{bmatrix} 1+3i \\ 2 \end{bmatrix}$$

$$\overline{A v_1} = \overline{\lambda_1 v_1}$$

property of complex conjugation is "the conjugate of a product is the product of the conjugates."

$$\overline{A}\,\overline{v_1} = \overline{\lambda_1}\,\overline{v_1}$$

Since $A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 2}$ then $\overline{A} = \begin{bmatrix} \overline{1} & \overline{5} \\ \overline{-2} & \overline{3} \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} = A$.

$$\boxed{A\,\overline{v_1} = \overline{\lambda_1}\,\overline{v_1}}$$

since $A = \overline{A}$.

this says $\overline{v_1}$ is an eigenvector for $\overline{\lambda_1} = \lambda_2$

$$A\overline{v_1} = \lambda_2 \overline{v_1}$$

thus an eigenvector for λ_2 is $\overline{v_1} = \overline{\begin{bmatrix} 1+3i \\ 2 \end{bmatrix}} = \begin{bmatrix} 1-3i \\ 2 \end{bmatrix}$