This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.

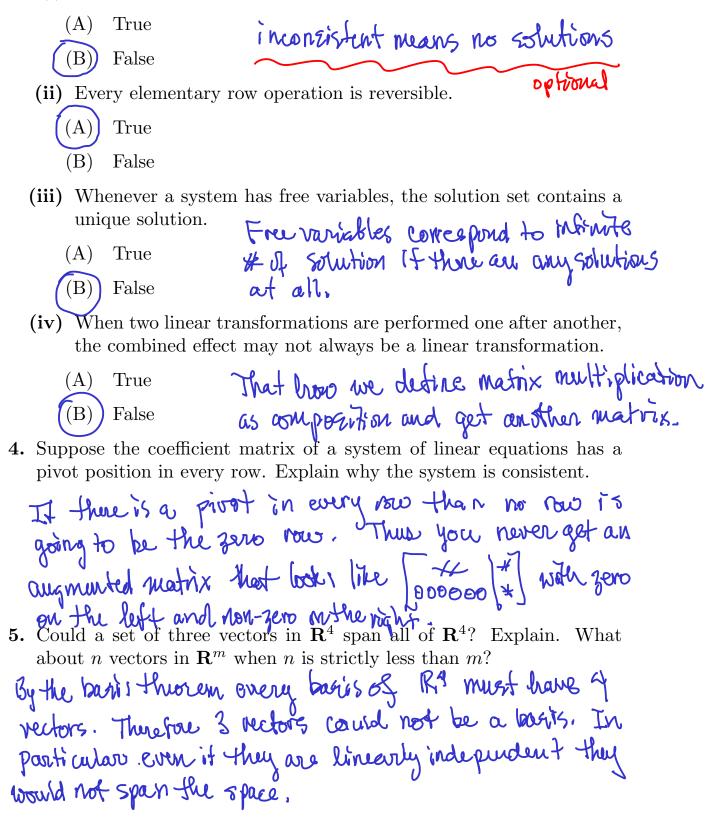
2. Suppose $u, v \in \mathbf{R}^3$ and $A \in \mathbf{R}^{2 \times 3}$ are given by

$$u = \begin{bmatrix} 2\\ -1\\ 3 \end{bmatrix}, \quad v = \begin{bmatrix} 1\\ 2\\ -3 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 2 & -1\\ 2 & 0 & -2 \end{bmatrix}.$$

(ii) Find Au.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 + 0 & -6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$

- **3.** Answer the following true false questions:
 - (i) An inconsistent system has more than one solution.



affernative solution to #5 fligt doesn't use 2.8 or 2.9.

Put the 3 rectors as columns in the Matrix A. This AER^{4×3}. Therectors span R^A if for every bER⁴ it is possible to solve Asc=b. Since A has more rows than columns, the echelor form of A will not have a pirot in every row. Thus at least one now of the echelon form is the zero row, This means I can't solve Arc=b for all right hand gides. Thus the vectors don't spar. In the general case AER Where m>n, again The new echebor form has at least one zero vow because thur are more vows than columns. It follows that the

commune of A doit span R^m.

Since there are 3 columns there can be at most 3 pivots. Thursfore one now doesn't have a pivot and muigt be the zero row,

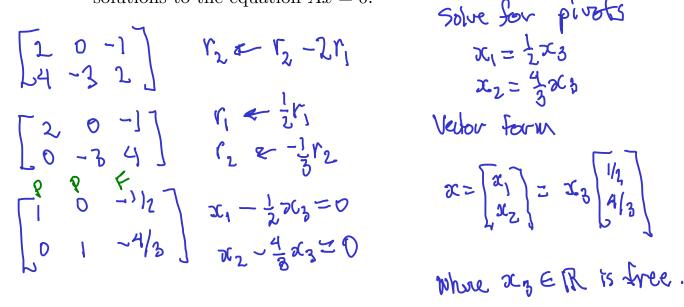
6. Write down the augmented matrix $\begin{bmatrix} A \mid b \end{bmatrix}$ corresponding to the system of linear equations given by $\begin{cases} 2x_1 + 4x_2 + 5x_3 - x_4 = 1 \\ 3x_1 + x_2 + 5x_3 - 4x_4 = 2 \\ -21x_1 + 5x_3 + 14x_4 = -7 \end{cases}$

but *do not* solve these equations.

7. Suppose $A \in \mathbb{R}^{2 \times 3}$ is given by

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix}.$$

How many free variables does the equation Ax = 0 have? Find all solutions to the equation Ax = 0.



8. The LU factorization of a matrix A is given by

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & -1 & 2 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{percherics preases}$$

Explain how to use this factorization to solve the equation Ax = b and then find the value of x corresponding to b = (1, 0, 4).

Since
$$Ax = b$$
 and $A = LU$ then $LUx = b$. Now
set $y \ge U_x$ and obtain $Ly = b$. Therefore, to find x .
Solve the system of systems $\begin{cases} Ly = b \\ Ux = y \end{cases}$.
Solve $hy = b$ first by substritution
 $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ -3 \\ -3 \\ -3 \\ y_1 + y_2 = 0 \\ y_3 = 4 + y_2 - 4 \\ y_1 = 4 + 3 - 4 = 3 \\ Thus \quad y = (1,3,3).$
Next solve $Ux = y$ by back substribution
 $x_3 = 3$.
 $x_3 = 3$.

$$\begin{array}{c} 2 - 1 & 1 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{array} \begin{array}{c} 3 \\ x_3 \\ z_3 \end{array} = \begin{array}{c} 1 \\ 3 \\ 3 \\ 3 \end{array} \begin{array}{c} -3 \\ x_1 - x_2 + 2x_3 = 3 \\ 3 \\ x_3 - 3 \\ x_1 - x_2 + 2x_3 = 1 \end{array}$$

$$x_3 = 3$$

 $x_2 = 3 - 4x_3 = -9$
 $-3 = 3$

$$x_1 = \frac{1 + x_2 - 2x_3}{2} = \frac{1 + 3 - 6}{2} = \frac{-2}{2} = -1$$

Solution
$$\mathcal{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$