Math 330: Sample Exam 2 Version G Exam

This is a closed-book closed-notes no-calculator-allowed in-class exam. Efforts have been made to keep the arithmetic simple. If it turns out to be complicated, that's either because I made a mistake or you did. In either case, do the best you can and check your work where possible. While getting the right answer is nice, this is not an arithmetic test. It's more important to clearly explain what you did and what you know.

1. Indicate in writing that you have understood the requirement to work independently by writing "I have worked independently on this quiz" followed by your signature as the answer to this question.

2. Consider the matrix A with reduced row eschelon form R where

$$A = \begin{bmatrix} -3 & 6 & 0 & 1 & 3 \\ -2 & 4 & 3 & 2 & 5 \\ -2 & 4 & 1 & 1 & 4 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 1 & -2 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & -9 \end{bmatrix}.$$

(i) Find a basis for $\mathrm{Col}(A)$.

(ii) Find a basis for $\mathrm{Nul}(A)$.

3. Answer the following true false questions:

(i) If A is invertible then $\det(A^{-1})(\det A) = 1$.

(A) True

(B) False

(ii) If A is invertible, then 0 is not an eigenvalue of A.

(A) True

(B) False

(iii) If $A \in \mathbf{R}^{n\times n}$ is triangular, then $\det A$ is the product of the diagonal entries of A.

(A) True

(B) False

(iv) The matrices A and $B^{-1}AB$ have the same sets of eigenvalues for every invertible matrix B.

(A) True

(B) False

4. Let λ be an eigenvalue of an invertible matrix A. Show λ^{-1} is an eigenvalue of A^{-1}.

5. What is the rank of a 5×7 matrix whose null space is two dimensional?

6. Find $\det(A)$, $\det(B)$ and $\det(AB)$ where

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -2 & 0 \\ 1/2 & 2/3 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

7. The matrix A given by

$$A = \begin{bmatrix} 12.6 & 2.2 & 8.6 \\ 9.6 & 3.2 & 6.6 \\ -16.8 & -3.6 & -11.8 \end{bmatrix}$$

has eigenvalues λ_i and eigenvectors x_i given by

$$\lambda_1 = 2, \quad x_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \qquad \lambda_2 = 3, \quad x_2 = \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}, \qquad \lambda_3 = -1, \quad x_3 = \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}.$$

Find an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$.

(i) What is D?

(ii) What is S?

8. Find the eigenvalues and eigenvectors of the matrix A where

$$A = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix}.$$