Written Homework 3

- Section 2.3 # 36

36. Show that if AB is invertible, so is B.

We assume both A and B are square $n \times n$ matrices. Since AB is invertible, then $(AB)^{-1}$ exists.

Claim $C = (AB)^{-1}A$ is the inverse of B. To check this, it is enough by part j of the invertible matrix theorem to show $CB = I$. Since

$$CB = (AB)^{-1}AB = (AB)^{-1}(AB) = I$$

then $CB = I$. Thus B is invertible and $B^{-1} = (AB)^{-1}A$.

- Section 2.8 # 43

43. What can you say about Nul B when B is a 5×4 matrix with linearly independent columns?

Since B has linearly independent columns, the the echelon form of B has a pivot in each column. This means the rank $B = \dim \operatorname{col} B = 4$. Moreover, there are no free variables — this observation is also called the rank theorem. Thus, $\dim \operatorname{Nul} B = 0$. It follows that $B = \{0\}$ is the trivial subspace consisting of only the zero vector.

- Section 2.9 # 28

28. What is the rank of a 4×5 matrix whose null space is three-dimensional?

Let A be the 4×5 matrix with $\dim \mathrm{Nul}\, A = 3$. By the rank theorem

$$\mathrm{rank}\, A + \dim \mathrm{Nul}\, A = n.$$

Since A has $n=5$ columns. Then $\mathrm{rank}\, A + 3 = 5$ implies $\mathrm{rank}\, A = 2$.