Math 430/630 Quiz 3 Version A

**1.** A list of vectors is called orthonormal if and only if

(A) the vectors in it are pairwise orthogonal and each vector has norm 1.

(B) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\langle v, e_k \rangle = 0$.

(C) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\langle v, e_k \rangle = 1$.

(D) for every $v \in V$ there exists $k \in \mathbf{N}$ such that $\|v\| = \|e_k\|$.

(E) none of these.

**2.** An orthonormal basis of $V$ is

(A) any list of linearly independent vectors $(e_1, \ldots, e_n)$ that span $V$.

(B) equal to range$(Q)$ where $Q \in \mathcal{L}(V)$ is an orthogonal operator.

(C) obtained by setting $e_i = v_i/\|v_i\|$ where $(v_1, \ldots, v_n)$ is a basis of $V$.

(D) an orthonormal list of vectors in $V$ that is also a basis of $V$.

(E) none of these.

**3.** The orthogonal complement of $U$, denoted $U^\perp$, is given by

(A) $U^\perp = \{\, v \in V :\ \text{there exists } u \in U \text{ such that } \langle v, u \rangle = 0 \,\}$.

(B) $U^\perp = \{\, v \in V :\ \text{there exists } u \in U \text{ such that } \langle v, u \rangle = 1 \,\}$.

(C) $U^\perp = \{\, v \in V : \langle v, u \rangle = 0 \text{ for all } u \in U \,\}$.

(D) $U^\perp = \{\, v \in V : \langle v, u \rangle = 1 \text{ for all } u \in U \,\}$.

(E) none of these.

**4.** Let $P$ be the orthogonal projection of $V$ onto $U$. Given $v \in V$ let $v = u + w$ where $u \in U$ and $w \in U^\perp$. Then generally

(A) $Pv = w$.

(B) $Pu = v$.

(C) $Pv = u$.

(D) $Pu = w$.

(E) none of these.

**5.** Let $V$ and $W$ be finite-dimensional complex inner product spaces and $T \in \mathcal{L}(V, W)$. The adjoint of $T$, denoted $T^*$, is the unique linear map in $\mathcal{L}(W, V)$ such that

(A) $\langle Tv, w \rangle = \langle T^*w, v \rangle$ for every $v \in V$ and $w \in W$.

(B) $\langle Tv, w \rangle = \langle v, T^*w \rangle$ for every $v \in V$ and $w \in W$.

(C) $TT^*w = w$ for every $w \in W$.

(D) $T^*Tv = v$ for every $v \in V$.

(E) none of these.

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**6.** Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}.$$

Find an orthogonal matrix $Q$ and an upper triangular matrix $R$ such that $A = QR$.

**7.** Prove one of following:

**Corollary 6.33:** If $U$ is a subspace of $V$, then $U = (U^{\perp})^{\perp}$.

**Adjoint of ajoint:** If $T \in \mathcal{L}(V, W)$, then $T = (T^*)^*$.

**8.** Extra Credit: Prove

**Triangle Inequality:** If $u, v \in V$ then $\|u + v\| \leq \|u\| + \|v\|$.