Math/CS 466/666 Fall 2007 Quiz 1

1. The computer codes

```
t=a+b
r1=t+c
```

and

```
t=b+c
r2=a+t
```

are mathematically equivalent and, if executed on a modern digital computer that implements the IEEE 754 standard for floating point arithmetic, will always result in the same value being assigned to the variables `r1` and `r2`.

(A) True

(B) False

2. Bound the error in the approximation $\cos(x) = 1 - \frac{1}{2}x^2$ for $-0.2 \le x \le 0.2$.

3. About how many decimal digits of precision do IEEE 754 single precision 4-byte floating point numbers have?

(A) 4

(B) 7

(C) 15

(D) 64

4. About how many decimal digits of precision do IEEE 754 double precision 8-byte floating point numbers have?

(A) 4

(B) 7

(C) 15

(D) 64

5. Given $p(x) = 4.5x^4 - 8.9x^3 + 5.1x^2 + 2.3x + 1.8$ explain how to calculate $p(1.3)$ using no more than 4 multiplications and 4 additions. Be as specific as possible.

6. Give a brief explanation of the following sources of error:

(i) Modeling Errors.

(ii) Physical Measurement Errors.

(iii) Machine Representation and Arithemetic Errors.

(iv) Mathematical Approximation Errors.

7. Find the 4th degree Taylor's polynomal for the function $f(x) = \dfrac{\sin(x)}{1-x}$.

8. Taylor's theorem states that $e^x = T_n(x) + R_n(x)$ where

$$T_n(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} \qquad \text{and} \qquad R_n(x) = \frac{x^{n+1}}{(n+1)!}e^c$$

with c between 0 and x. Use the remainder term $R_n(x)$ to find a value of n that guarantees $|T_n(1/2) - \sqrt{e}| \leq 10^{-7}$.