Exam II Review

Mon Nov 17 11:30:54 PST 2008 Version 1

Please know the following for the exam Friday, Nov 21.

1. **Be able to do all problems from the quizes and homework.**
   a. Note that some homework problems are too lengthy or computational for an exam question. Please understand these homework problems anyway, as I may simplify them for the exam.
2. Taylor's Theorem.
   a. Know how to create the Taylor polynomial.
   b. Know the remainder term.
   c. Know how to use the remainder term to estimate errors.
3. Polynomial evaluation.
   a. Know how to use nested multiplication and synthetic division.
   b. How many multiplications and additions are required to evaluate a typical n-th degree polynomial?
4. Root Finding.
   a. State the bisection method, Newton's method and the secant method.
   b. Compare the advantages and disadvantages of each of these three methods.
   c. Be able to predict how many iterations of bisection method are necessary to find a root to a prescribed accuracy.
   d. Show that Newton's method is quadratically convergent.
   e. How to use Newton's method to find roots of multiplicity.
5. Polynomial Interpolation.
   a. Definition of the Lagrange basis functions.
   b. Definition of divided differences.
   c. Statement of Theorem 4.2.1 for the error in polynomial interpolation.
   d. Definition and properties of Chebyshev polynomials.
   e. Use of Chebyshev polynomials in finding near min-max polynomial.
   f. Definition and properties of Legendre polynomials.
   g. Use of Legendre polynomials in finding least squares approximations.

6. Numerical Quadrature
   a. State the trapezoid method and the Simpson's methods.
   b. Know that the error in the trapezoid method decreases as $O(h^2)$ as $h \to 0$.
   c. Know that the error in Simpson's method decreases as $O(h^4)$ as $h \to 0$.
   d. Let E be the error and $h = (b-a)/n$ be the space between each of the nodal points x_i. Explain how to numerically verify that Simpson's method is working from a graph of $\log |E|$ versus $\log h$.
   e. Know that a Gaussian quadrature with n nodal points will integrate polynomials of degree $2n-1$ exactly.
   f. Explain the idea behind Gaussian quadrature. In particular, why would a quadrature formula that integrates high degree polynomials exactly approximate the integral of other functions well?
   g. Know how to rescale a definite integral on the interval $[a, b]$ to the interval $[-1, 1]$ by changing variables.
   h. Explain how to find the nodal points x_i in the Gaussian quadrature formula using the Legendre polynomials.
   i. Explain how to find the weights w_i in the Gaussian quadrature formula given the nodal points x_i.
7. Psuedocode
   a. To efficiently evaluate $p(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$ given a value of x and an array a_i for the coefficients.
   b. To implement bisection, Newton and secant methods.
   c. To implement trapezoid and Simpson's methods.
8. Numerical Differentiation
   a. Be able to analyse the approximation $f'(x) \approx \big(f(x+h) - f(x-h)\big)/2h$ using Taylor's theorem.
   b. Explain how roundoff error implies a minimum optimal size for h and how significant digits are lost by the subtraction of two nearly equal numbers in the numerator when performing numerical differentiation.
9. Understand and be able to reproduce the examples from the book:
   a. Example 1.2.4
   b. Taylor series example on page 49.
   c. Given $x_0, \ldots, x_n$ and $f(x_0), \ldots, f(x_n)$ be able to compute divided differences $f(x_0, x_1)$, $f(x_1, x_2)$, $f(x_0, x_1, x_2)$ and so forth. Use these divided differences in formulae (4.33)–(4.35) on page 129 to find interpolating polynomials.
   d. Given the table of weights w_i and nodal points x_i be able to use a 2 or 3 point Gaussian quadrature formula to approximate a definite integral such as Example 5.3.2 and 5.3.4.