```
> restart;
  dp:=(p,q)->int(p*q,x=-1..1);
  nm:=p->sqrt(dp(p,p));
```

$$dp := (p, q) \rightarrow \int_{-1}^{1} p\, q \, dx$$

$$nm := p \rightarrow \sqrt{dp(p, p)} \tag{1}$$

```
> N:=3;
```

$$N := 3 \tag{2}$$

```
> for n from 0 to N
  do
    tv[n]:=x^n-add(dp(x^n,tv[k])*tv[k],k=0..n-1);
    v[n]:=tv[n]/nm(tv[n]);
  end;
```

$$tv_0 := 1$$

$$v_0 := \frac{1}{2}\sqrt{2}$$

$$tv_1 := x$$

$$v_1 := \frac{1}{2} x \sqrt{6}$$

$$tv_2 := x^2 - \frac{2}{3}$$

$$v_2 := \frac{1}{2}\left(x^2 - \frac{2}{3}\right)\sqrt{10}$$

$$tv_3 := x^3 - \frac{2}{5} x$$

$$v_3 := \frac{5}{38}\left(x^3 - \frac{2}{5} x\right)\sqrt{798} \tag{3}$$

```
> xs:=[solve(v[3]=0,x)];
```

$$xs := \left[0, \frac{1}{5}\sqrt{10}, -\frac{1}{5}\sqrt{10}\right] \tag{4}$$

```
> ws:=[seq(w[k],k=0..2)];
```

$$ws := [w_0, w_1, w_2] \tag{5}$$

```
> q:=f->add(ws[k]*f(xs[k]),k=1..3);
```

$$q := f \rightarrow add(ws_k\, f(xs_k), k = 1..3) \tag{6}$$

```
> eq1:=int(1,x=-1..1)=q(x->1);
  eq2:=int(x,x=-1..1)=q(x->x);
  eq3:=int(x^2,x=-1..1)=q(x->x^2);
```

$$eq1 := 2 = w_0 + w_1 + w_2$$

$$eq2 := 0 = \frac{1}{5} w_1 \sqrt{10} - \frac{1}{5} w_2 \sqrt{10}$$

$$eq3 := \frac{2}{3} = \frac{2}{5} w_1 + \frac{2}{5} w_2 \tag{7}$$

```
> S:=solve({eq1,eq2,eq3},ws);
```

(8)

$$S := \left[\left[w_0 = \frac{1}{3}, w_1 = \frac{5}{6}, w_2 = \frac{5}{6} \right] \right] \quad (8)$$

```
> a1:=subs(S[1],q(f));
```

$$a1 := \frac{1}{3} f(0) + \frac{5}{6} f\left(\frac{1}{5} \sqrt{10} \right) + \frac{5}{6} f\left(-\frac{1}{5} \sqrt{10} \right) \quad (9)$$

```
> with(codegen):
> C(a1);
    t0 = f(0.0)/3.0+5.0/6.0*f(sqrt(10.0)/5.0)+5.0/6.0*f(-sqrt
(10.0)/5.0);
> solve((x-b)/(b-a)+(x-a)/(b-a)=u,x);
```

$$-\frac{1}{2} u a + \frac{1}{2} u b + \frac{1}{2} a + \frac{1}{2} b \quad (10)$$

```
> evalf(int(exp(-x^2),x=-1..1));
```

$$1.493648266 \quad (11)$$