Math/CS 466/666 Programming Project/Homework 3

Newton's Method

Your work should be presented in the form of a typed report using clear and properly
punctuated English. Where appropriate include full program listings and output. If
you choose to work in a group of two, please turn in independently prepared reports.

Consider Newton's method for solving $f(x) = 0$ where $f(x) = x^3 - 3$ using the starting
point $x_0 = 1$.

1. Let $e_n = x_n - 3^{1/3}$ and create a table with three columns showing n, x_n and e_n for
   $n = 0, 1, \ldots, 8$.

2. A sign of quadratic convergence is that the number of significant digits double at each
   iteration. Does that happen in this case?

3. Comment on how rounding error effects the convergence of Newton's method.

4. Write $|e_{n+1}| = M_n |e_n|^2$ and compute M_n for $n = 1, 2, 3$, and 4. In this case is M_n
   bigger or less than 1?

5. Find the limit of M_n when $n \to \infty$ analytically. What is the exact value of the limit?