

I wrote a program called flops.c that solves a system of linear equations $Ax = b$. Note, the solution obtained by Gaussian elimination satisfies

$$b_i = \sum_{j=1}^n A_{ij}x_j.$$

The program was this:

```

1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4  #include <math.h>
5  #include "tictoc.h"
6
7  void scale(int n,double v[],double alpha){
8      for(int i=0;i<n;i++){
9          v[i]*=alpha;
10     }
11 }
12 void eliminate(int n,double v1[],
13     double v2[],double alpha){
14     for(int i=0;i<n;i++){
15         v1[i]+=alpha*v2[i];
16     }
17 }
18 void swap(int n,double v1[],double v2[]){
19     for(int i=0;i<n;i++){
20         double t=v1[i]; v1[i]=v2[i]; v2[i]=t;
21     }
22 }
23 void matprint(int n,double A[n][n]){
24     for(int i=0;i<n;i++){
25         for(int j=0;j<n;j++){
26             printf("%g ",A[i][j]);
27         }
28         printf("\n");
29     }
30 }
31 // Factor A into PLU where L is lower triangular with 1's on
32 // the diagonal and U is upper triangular. Return the result
33 // in A where the upper triangular part of A is now U and the
34 // part of A below the diagonal holds L.
35 void lufact(int n,double A[n][n],int P[n]){
36     for(int j=0;j<n;j++){
37         P[j]=j;
38     }
39     for(int j=0;j<n-1;j++){
40         // Search for the largest pivot in the column

```

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41         int i0=j;
42         for(int i=j+1;i<n;i++){
43             if(fabs(A[i][j])>fabs(A[i0][j])){
44                 i0=i;
45             }
46         }
47         swap(n,A[j],A[i0]);
48         int t=P[j]; P[j]=P[i0]; P[i0]=t;
49         for(int i=j+1;i<n;i++){
50             double alpha=-A[i][j]/A[j][j];
51             eliminate(n-j-1,A[i]+j+1,A[j]+j+1,alpha);
52             A[i][j]=-alpha;
53         }
54     }
55 }
56 // Solve PLUX=b for x where LU is the matrix returned by lufact.
57 void lusolve(int n,int P[n],double LU[n][n],double x[n],double b[n]){
58     double y[n];
59     // Forward substitute to solve Ly=b
60     for(int i=0;i<n;i++){
61         y[i]=b[P[i]];
62         for(int j=0;j<i;j++){
63             y[i]-=LU[i][j]*y[j];
64         }
65     }
66     // Backward substitute to solve Ux=y
67     for(int i=n-1;i>=0;i--){
68         x[i]=y[i];
69         for(int j=i+1;j<n;j++){
70             x[i]-=LU[i][j]*x[j];
71         }
72         x[i]/=LU[i][i];
73     }
74 }
75 // Compute b where b=Ax.
76 void multAb(int n,double b[n],double A[n][n],double x[n]){
77     for(int i=0;i<n;i++){
78         b[i]=0;
79         for(int j=0;j<n;j++){
80             b[i]+=A[i][j]*x[j];
81         }
82     }
83 }
84 // Compute the norm |x-y|.
85 double dist(int n,double x[n],double y[n]){
86     double s=0.0;
87     for(int i=0;i<n;i++){
88         double t=x[i]-y[i];
89         s+=t*t;

```

```

90     }
91     return sqrt(s);
92 }
93
94 #define N 1000
95 double A[N][N];
96 double b[N];
97 double x[N], LU[N][N], b2[N];
98 int P[N];
99
100 int main(){
101     for(int i=0; i<N; i++){
102         for(int j=0; j<N; j++){
103             A[i][j]=2.0*random()/RAND_MAX-1.0;
104         }
105         b[i]=2.0*random()/RAND_MAX-1.0;
106     }
107     memcpy(LU, A, N*N*sizeof(double));
108     tic();
109     lufact(N, LU, P);
110     double tfact=toc();
111     tic();
112     lusolve(N, P, LU, x, b);
113     double tsolve=toc();
114     multAb(N, b2, A, x);
115     printf("N=%d\n", N);
116     printf("error=%.15e\n", dist(N, b, b2));
117     printf("tfact=%g sec\n", tfact);
118     printf("tsolve=%g sec\n", tsolve);
119     printf("total time=%g sec\n", tfact+tsolve);
120     printf("FLOPS=%g\n", 2.0*(N/3.0+1.0)*N*N/(tfact+tsolve));
121     return 0;
122 }
123

```

The output was this:

```

N=1000
error=2.119188909422022e-11
tfact=0.633473 sec
tsolve=0.00241295 sec
total time=0.635886 sec
FLOPS=1.05155e+09

```