Iterative Scheme Example

Want to solve the polynomial equation

$$p(x) = 0$$

In [2]:
```
p(x)=((3*x+2)*x+1)*x+1;
```

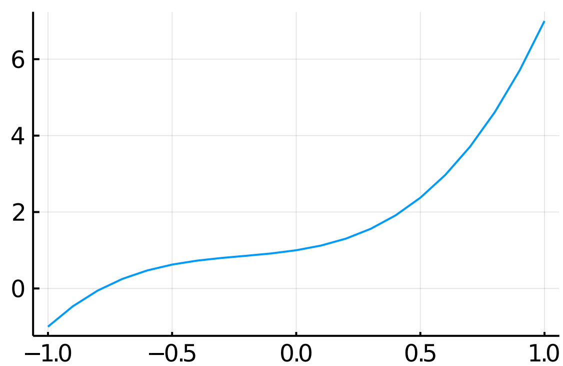
In [3]:
```
xs=[-1:0.1:1;];
```

In [4]:
```
ys=p.(xs);
```

In [5]:
```
using Plots
```

In [6]:
```
plot(xs,ys,size=[300,200],legend=false)
```

Out[6]:



From the graph it appears the root $\alpha \approx 0.75$ and somewhere between 0.7 and 0.8.

Solve $3x^3 + 2x^2 + x + 1 = 0$ for x in a simple way to obtain $x = F(x)$ where

$$F(x) = -3x^3 - 2x^2 - 1.$$

Now check whether $|F'(x)| < 1$ near the root α.

In [7]:
```
dF(x)=-(9*x+4)*x;
```

In [8]:
```
dF(-0.75)
```

Out[8]:
```
-2.0625
```

In [10]:
```
dF(-0.8)
```

Out[10]: -2.5600000000000005

In [12]:
```
dF(-0.7)
```

Out[12]: -1.6099999999999999

As none of these values for $F'(x)$ have magnitude less than 1 the iterative scheme $x_{n+1} = F(x_n)$ is doomed. However, it is possible to use the simple trick of adding mx to both sides to obtain

$$3x^3 + 2x^2 + (m+1)x + 1 = mx$$

and dividing so that now $x = F(x)$ where

$$F(x) = \frac{-3x^3 - 2x^2 + mx - 1}{m+1}.$$

In [13]:
```
m=10
dF(x)=(-(9*x+4)*x+m)/(m+1);
```

In [14]:
```
dF(-0.75)
```

Out[14]: 0.7215909090909091

In [15]:
```
dF(-0.8)
```

Out[15]: 0.6763636363636363

In [17]:
```
dF(-0.7)
```

Out[17]: 0.7627272727272728

Note that $|F'(x)| < 1$ for the new F in a neighborhood of the root.

Define the iteration

$$x_{n+1} = F(x_n)$$

and check numerically how it converges.

In [20]:

```
F(x)=(((-3*x-2)*x+m)*x-1)/(m+1)
```

Out[20]:

```
F (generic function with 1 method)
```

In [23]:

```
x=-0.75
for n=1:20
    x=F(x)
    println(n," ",x)
end
```

```
1 -0.7599431818181818
2 -0.7670751308110003
3 -0.7721371822812876
4 -0.7757025980518968
5 -0.7782000428150956
6 -0.779942560069474
7 -0.781154989966631
8 -0.7819969549081353
9 -0.782580861755282
10 -0.7829854227796318
11 -0.7832655403700722
12 -0.7834594054810547
13 -0.783593534388825
14 -0.7836863135772657
15 -0.7837504808103654
16 -0.7837948550353727
17 -0.7838255393856315
18 -0.7838467562608255
19 -0.7838614262884921
20 -0.7838715693739893
```

It's slowly converging to $\alpha \approx -0.7838\ldots$. The Aitkin's δ^2 process will be used next time to accelerate this convergence using extrapolation.

In [ ]: