	x	5D $f(x) = e^x$	Differences 1st.	2nd.	3rd.
1	0.10	1.10517			
			means .01111 1111		
2	0.11	1.11628		11	
			1122		0
3	0.12	1.12750		11	
			1133		0
4	0.13	1.13883		11	
			1144		1
5	0.14	1.15027		12	
			1156		0
6	0.15	1.16183		12	
			1168		−1
7	0.16	1.17351		11	
			1179		2
8	0.17	1.18530		13	
			1192		
9	0.18	1.19722			

```
julia> 111628-110517
1111
```

```julia
f(x)=exp(x)
xs=0.1:0.01:0.18
for x in xs
    println(x)
end
```

```
julia> include("table.jl")
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
```

```julia
xs=0.1:0.01:0.18
```

This is an iterator and length(xs) is the number of iterations it stands for...

```julia
f(x)=exp(x)
xs=0.1:0.01:0.18
N=length(xs)
M=zeros(N,N)
for i=1:N
    M[i,1]=f(xs[i])
end
```

```
julia> include("table.jl")

julia> M
9×9 Matrix{Float64}:
 1.10517  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.11628  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.1275   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.13883  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.15027  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.16183  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.17351  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.1853   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 1.19722  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```
julia> for i=1:N-1
               M[i,2]=M[i+1,1]-M[i,1]
       end

julia> M
9×9 Matrix{Float64}:
 1.10517  0.0111072  0.0  0.0  0.0  0.0
 1.11628  0.0112188  0.0  0.0  0.0  0.0
 1.1275   0.0113315  0.0  0.0  0.0  0.0
 1.13883  0.0114454  0.0  0.0  0.0  0.0
 1.15027  0.0115604  0.0  0.0  0.0  0.0
 1.16183  0.0116766  0.0  0.0  0.0  0.0
 1.17351  0.011794   0.0  0.0  0.0  0.0
 1.1853   0.0119125  0.0  0.0  0.0  0.0
 1.19722  0.0        0.0  0.0  0.0  0.0
```

First column of differences...

```
julia> for i=1:N-2
           M[i,3]=M[i+1,2]-M[i,2]
       end

julia> M
9×9 Matrix{Float64}:
 1.10517   0.0111072   0.000111629
 1.11628   0.0112188   0.000112751
 1.1275    0.0113315   0.000113884
 1.13883   0.0114454   0.000115028
 1.15027   0.0115604   0.000116184
 1.16183   0.0116766   0.000117352
 1.17351   0.011794    0.000118531
 1.1853    0.0119125   0.0
 1.19722   0.0         0.0
```

2nd differences...

rounded to 5D

x	5D $f(x) = e^x$	*Differences* 1st.	2nd.	3rd.
0.10	1.10517			
		1111		
0.11	1.11628		11	
		1122		0
0.12	1.12750		11	
		1133		0
0.13	1.13883		11	
		1144		1
0.14	1.15027		12	
		1156		0
0.15	1.16183		12	
		1168		−1
0.16	1.17351		11	
		1179		2
0.17	1.18530		13	
		1192		
0.18	1.19722			

different

Done using 64-bit floating point

≈ 15S

significant digits

```julia
f(x)=exp(x)
xs=0.1:0.01:0.18
N=length(xs)
M=zeros(N,N)
for i=1:N
    M[i,1]=f(xs[i])
end
for i=1:N-1
    M[i,2]=M[i+1,1]-M[i,1]
end
for i=1:N-2
    M[i,3]=M[i+1,2]-M[i,2]
end
```

spacing

equally spaced values for x

```julia
f(x)=exp(x)
xs=0.1:0.01:0.18
N=length(xs)
M=zeros(N,N)
for i=1:N
    M[i,1]=f(xs[i])
end
for j=2:N
    for i=1:N+1-j
        M[i,j]=M[i+1,j-1]-M[i,j-1]
    end
end
```


```
0.000111629
0.000112751
0.000113884
0.000115028
0.000116184
0.000117352
0.000118531
0.0
0.0
```


```
julia> include("table.jl")
```

```
julia> M
9×9 Matrix{Float64}:
 1.10517   0.0111072   0.000111629   1.12189e-6
 1.11628   0.0112188   0.000112751   1.13316e-6
 1.1275    0.0113315   0.000113884   1.14455e-6
 1.13883   0.0114454   0.000115028   1.15605e-6
 1.15027   0.0115604   0.000116184   1.16767e-6
 1.16183   0.0116766   0.000117352   1.17941e-6
 1.17351   0.011794    0.000118531   0.0
 1.1853    0.0119125   0.0           0.0
 1.19722   0.0           0.0           0.0
```

```julia
1 f(x)=x^2-3*x+5
```

```
julia> include("table.jl")
p (generic function with 1 method)

julia> M
9×9 Matrix{Float64}:
 4.71    -0.0279  0.0002   8.88178e-16  -1.77636e-15   3.55271e-15  -6.21725e-15   8.88178e-15  -9.76996e-15
 4.6821  -0.0277  0.0002  -8.88178e-16   1.77636e-15  -2.66454e-15   2.66454e-15  -8.88178e-16   0.0
 4.6544  -0.0275  0.0002   8.88178e-16  -8.88178e-16   0.0          1.77636e-15   0.0          0.0
 4.6269  -0.0273  0.0002   0.0          -8.88178e-16   1.77636e-15   0.0          0.0          0.0
 4.5996  -0.0271  0.0002  -8.88178e-16   8.88178e-16   0.0          0.0          0.0          0.0
 4.5725  -0.0269  0.0002   0.0           0.0          0.0          0.0          0.0          0.0
 4.5456  -0.0267  0.0002   0.0           0.0          0.0          0.0          0.0          0.0
 4.5189  -0.0265  0.0      0.0           0.0          0.0          0.0          0.0          0.0
 4.4924   0.0     0.0      0.0           0.0          0.0          0.0          0.0          0.0
```

1st diff. 2nd diff 3rd diff

zero to within rounding error.

all zero

rounding errors propagated

If $f(x)$ is a polynomial of degree n then the nth differences are constant and the $(n+1)$-th differences are zero... except for rounding errors...

Given a table of numbers and the desire to approximate it with a polynomial... choose the smallest degree polynomial so that the error left is explained by rounding errors...

	Differences					
Tabular error	*1st.*	*2nd.*	*3rd.*	*4th.*	*5th.*	*6th.*
$+\frac{1}{2}$						
	-1					
$-\frac{1}{2}$		$+2$				
	$+1$		-4			
$+\frac{1}{2}$		-2		$+8$		
	-1		$+4$		-16	
$-\frac{1}{2}$		$+2$		-8		$+32$
	$+1$		-4		$+16$	
$+\frac{1}{2}$		-2		$+8$		
	-1		$+4$			
$-\frac{1}{2}$		$+2$				
	$+1$					
$+\frac{1}{2}$						

worst case rounding errors..,

& most rounding error thus could have been in the table

A rough working criterion for the *expected* fluctuations ('noise level') due to round-off error is shown in the following table.

Order of difference	1	2	3	4	5	6
Expected error limits	±1	±2	±3	±6	±12	±22

sometimes the errors cancel ... so on average one excepts less rounding error

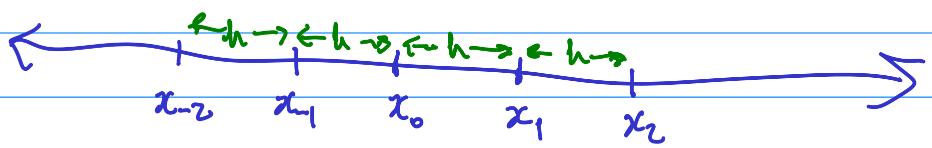
1 The shift operator E

2 The forward difference operator Δ

3 The backward difference operator ∇

4 The central difference operator δ

Let $h > 0$ $\quad x_i = x_0 + ih$ ← grid of x values



Let $f(x)$ be a function $\quad f_i = f(x_i)$

values of f at grid points...

relates the value of f at one place to another

(1) $Ef_i = f_{i+1}$

$$E^2 f_i = EEf_i = Ef_{i+1} = f_{i+2}$$

(2) $\Delta f_i = Ef_i - f_i = (E - I)f_i = f_{i+1} - f_i$

(3) $\nabla f_i = (I - E^{-1})f_i = f_i - f_{i-1}$