Notation:

$$f(x_0,x_1)=\frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$f(x_0,x_1,x_2)=\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$

$$f(x_0,x_1,x_2,x_3)=\frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$$

$$\vdots$$

$$f(x_0,\dots,x_n)=\frac{f(x_1,\dots,x_n)-f(x_0,\dots,x_{n-1})}{x_n-x_0}$$

n	x_n	$f(x_n)$	$f(x_n,x_{n+1})$	$f(x_n,x_{n+1},x_{n+2})$	$f(x_n,x_{n+1},x_{n+\cdot}$
0	x_0	$f(x_0)$	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	
1	x_1	$f(x_1)$	$f(x_1,x_2)$	$f(x_1,x_2,x_3)$	
2	x_2	$f(x_2)$	$f(x_2,x_3)$		
3	x_3	$f(x_3)$			

```
f(x)=1/(x^2+1)
xs=0:0.5:4
N=length(xs)
M=zeros(N,N)
for i=1:N
    M[i,1]=f(xs[i])
end
for j=2:N
    for i=1:N-j+1
        M[i,j]=(M[i+1,j-1]-M[i,j-1])/((xs[i+j-1]-xs[i]))
    end
end
```

no longer need to be equally spaced.

the same

need divided difference.

$$\frac{f(x_1)-f(x_0)}{x_1-x_0}$$
$$=\frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0}$$
$$x_3)=\frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$$
$$n)=\frac{f(x_1,\dots,x_n)-f(x_0,\dots,x_{n-1})}{x_n-x_0}$$

Table of differences

$f(x_n)$ $\quad f(x_n, x_{n+1})$ $\quad f(x_n, x_{n+1}, x_{n+2})$ $\quad f(x_n, x_{n+1}, x_{n+2}, x_{n+3})$ $\quad f(x_n, \ldots, x_{n+4})$

aren't that small...

```
julia> M
5×5 Matrix{Float64}:
 0.914549  -0.61449    0.13598     0.00268646  -0.0151367
 0.702407  -0.425864   0.142321   -0.0380682    0.0
 0.258686  -0.139078   0.0529661   0.0          0.0
 0.123344  -0.0699418  0.0         0.0          0.0
 0.100113   0.0        0.0         0.0          0.0
```

$$p(x) = f(x_0)\phi_0(x) + f(x_0,x_1)\phi_1(x) + f(x_0,x_1,x_2)\phi_2(x) + \cdots + f(x_0,\ldots,x_n)\phi_n(x)$$

Need to program these basis functions...

$$\phi_0(x) = 1$$
$$\phi_1(x) = x - x_0$$
$$\phi_2(x) = (x-x_0)(x-x_1)$$
$$\phi_3(x) = (x-x_0)(x-x_1)(x-x_2)$$
$$\vdots$$
$$\phi_n(x) = (x-x_0)\cdots(x-x_{n-1})$$

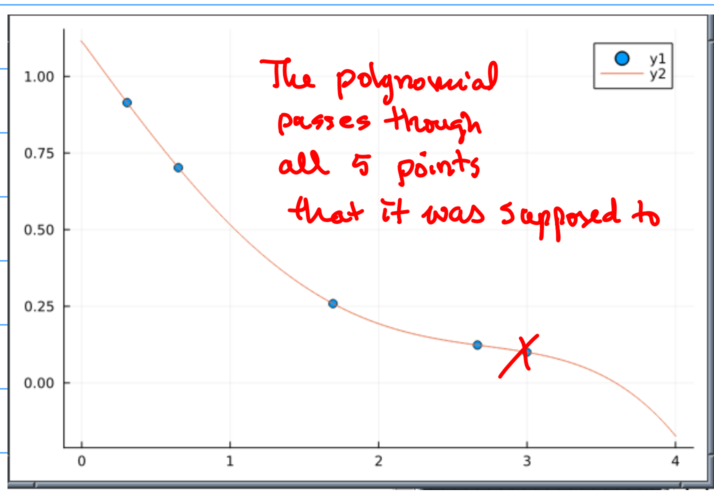
Code for basis functions...

```
function phi(n,x)
    r=1.0
    for k=1:n
        r*=(x-xs[k])
    end
    return r
end
```

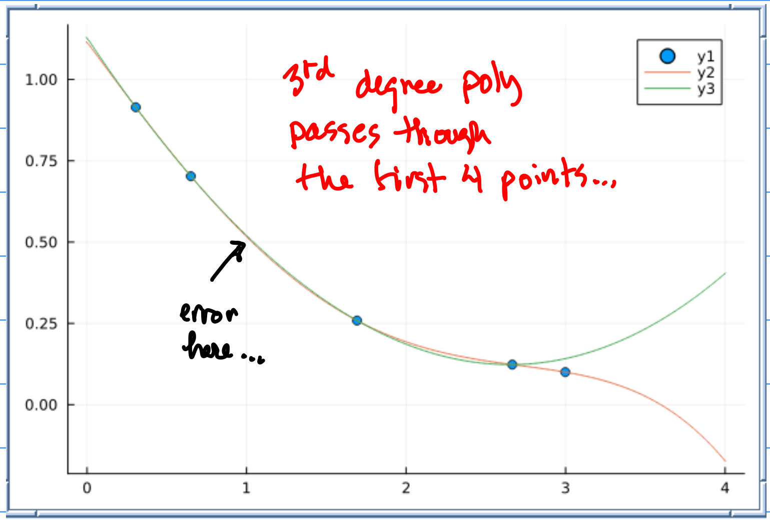
Polynomial of degree d can now be written...

```
function p(x,d)
    s=0
    for j=1:d+1
        s+=phi(j-1,x)*M[1,j]
    end
    return s
end
```

```
julia> xs2=0:0.01:4
0.0:0.01:4.0

julia> g(x)=p(x,4)
g (generic function with 1 method)

julia> plot!(xs2,g.(xs2))
```



```
julia> g3(x)=p(x,3)
g3 (generic function with 1 method)

julia> plot!(xs2,g3.(xs2))
```



Note to skip other point redefine what the xs are and recompute the difference table...

What is the error in the interpolation?

absolute error = $|p(1) - f(1)|$

$f(1) = \frac{1}{2}$

$P_4(1) =$ `0.5161647279202446`

$P_3(1) \approx$ `0.5203996008751721`

```
julia> f(1)
0.5

julia> g3(1)
0.5203996008751721

julia> g(1)
0.5161647279202446
```

Error in the 3rd degree interpolation

```
julia> abs(g3(1)-f(1))
0.02039960087517212
```

Error in the 4nd degree interpolation

```
julia> abs(g(1)-f(1))
0.016164727920244593
```

Read this over the long weekend (if you want)

Step 2)

5 Aitken's method

In practice, a procedure due to Aitken is often adopted, in which successively better interpolation polynomials (corresponding to successively higher order truncation of Newton's divided difference formula) are determined systematically. Thus, one has