```
$ cal
    December 2025
Su Mo Tu We Th Fr Sa
    1  2  3  4  5  6
 7  8  9 10 11 12 13
14 15 16 17 18 19 20
21 22 23 24 25 26 27
28 29 30 31
```

Homework 2 due...

Each solution a separate PDF file so its easier to compare different solutions to the same problem...

By the end of the day the link saying which problems to turn in should be active...

$x - \frac{f(x)}{f'(x)}$

```
correct=0
x=x0
println()
for i=1:3
    global x,correct
    x=newton(x);
    println("x_$(i) = $x")
    if abs(x-xngold[i])<1e-7
        correct+=1
    end
end
println()
```

```
#  Define any additional functions you need here
```

$f(x) = \log(x) - x + 2$; $df(x) = \frac{1}{x} - 1$

```
function newton(x)

    #  Please add lines to this function and fix
    #  the return value so it's not zero

    return
end
```

$x - f(x)/df(x)$

```
println("This is Problem 2 for Exam 2.")

xngold=[3.386294361119891,
    3.149938393803694,
    3.1461942570271373 ]
```

What's this?

```
if correct!=3
    println("Please try again.  Your answer is incorrect.")
else
    println("Congratulations!  Your answer is correct!")
end
```

Now move to next problem...

over all s

$$k_i = f\left(t_n + c_i h, y_n + h\sum_{j=1}^{s} a_{ij}k_j\right).$$

Each method listed on this page is defined by its Butche

the method in a table as follows:

$$\begin{array}{c|cccc} c_1 & a_{11} & a_{12} & \dots & a_{1s} \\ c_2 & a_{21} & a_{22} & \dots & a_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_s & a_{s1} & a_{s2} & \dots & a_{ss} \\ \hline & b_1 & b_2 & \dots & b_s \end{array}$$

$$\begin{array}{c|cc} c_1 & a_{11} & a_{12} \\ c_2 & a_{21} & a_{22} \\ \hline & b_1 & b_2 \end{array}$$

$s=2$ only nested once.

$$u(t_{i+1}) \approx u(t_i) + \frac{h}{2} f(t_i, u(t_i)) + \frac{h}{2} f(t_{i+1}, u(t_i) + h f(t_i, u(t_i)))$$

change this to equality and let $u_i \approx u(t_i)$

$t_i := t_0 + hi$

approximate solution that exactly satisfies

exact solution that satisfies the approximation

time step

$$u_{i+1} = u_i + \frac{h}{2} f(t_i, u_i) + \frac{h}{2} f(t_{i+1}, u_i + h f(t_i, u_i))$$

$$k_i = f\left(t_n + c_i h,\, y_n + h\sum_{j=1}^{s} a_{ij}k_j\right). \qquad y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i$$

$$u_{n+1} = u_n + \frac{h}{2} f(t_n, u_n) + \frac{h}{2} f(t_{n+1}, u_n + h f(t_n, u_n))$$

$$+\, h\Big(\underbrace{\tfrac{1}{2}}_{b_1=\frac{1}{2}} \underbrace{f(t_n, u_n)}_{k_1} + \underbrace{\tfrac{1}{2}}_{b_2=\frac{1}{2}} \underbrace{f(t_{n+1}, u_n + h f(t_n, u_n))}_{k_2}\Big)$$

$$k_1 = f(t_n + \underbrace{c_1}_{c_1=0} h,\, u_n + h(\underbrace{a_{11}}_{a_{11}=0} k_1 + \underbrace{a_{12}}_{a_{12}=0} k_2)) = f(t_n, u_n)$$

$$\begin{array}{c|cc} c_1 & a_{11} & a_{12} \\ c_2 & a_{21} & a_{22} \\ \hline & b_1 & b_2 \end{array} \qquad \begin{array}{c|cc} 0 & 0 & 0 \\ c_2 & a_{21} & a_{22} \\ \hline & 1/2 & 1/2 \end{array}$$

$t_n = t_0 + hn$

$t_{n+1} = t_0 + h(n+1) = t_n + h$

$$k_2 = f(\underbrace{t_n + c_2 h}_{c_2 = 1},\, u_n + h(\overbrace{\underbrace{a_{21}}_{a_{21}=1} k_1 + \underbrace{a_{22}}_{a_{22}=0} k_2}^{f(t_n, u_n)})) = f(\overbrace{t_{n+1}}^{t_n + h}, u_n + h f(t_n, u_n))$$

Butcher Tablea for the method from Lab 5 also called RK2 or IE2.

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & 1/2 & 1/2 \end{array}$$

Is this an RK method? YES What's the tableau?

$$u(t_{i+1}) - h\, f(t_{i+1}, u(t_{i+1})) \approx u(t_i)$$ approximation with exact solution.

$$u_{n+1} - h\, f(t_{n+1}, u_{n+1}) = u_n$$ exact equation defining the approximations

$$\begin{array}{c|c} 1 & 1 \\ \hline & 1 \end{array}$$

$$k_i = f\left(t_n + c_i h, y_n + h \sum_{j=1}^{s} a_{ij} k_j\right).$$

$$k_1 = f(\underbrace{t_n + c_1 h}_{}, \underbrace{y_n + h a_{11} k_1}_{\text{appox at } t_{n+1}}) = f(\underbrace{t_{n+1}}_{t_n + h}, u_{n+1})$$

$c_1 = 1$ $\qquad$ $a_{11} = 1$

is this the same?

$$k_1 = f(t_n + h, y_n + h k_1) = f(t_{n+1}, y_{n+1})$$

$$u_{n+1} = u_n + h\, f(t_{n+1}, u_{n+1})$$

$$y_{n+1} = y_n + \overbrace{h \sum_{i=1}^{s} b_i k_i} = y_n + h k_1$$

So implicitly solving for k_1 such that

$$k_1 = f(t_n + h, y_n + h k_1)$$

is equivalent to implicitly solving for y_{n+1} so that

$$y_{n+1} = y_n + h\, f(t_{n+1}, y_{n+1})$$