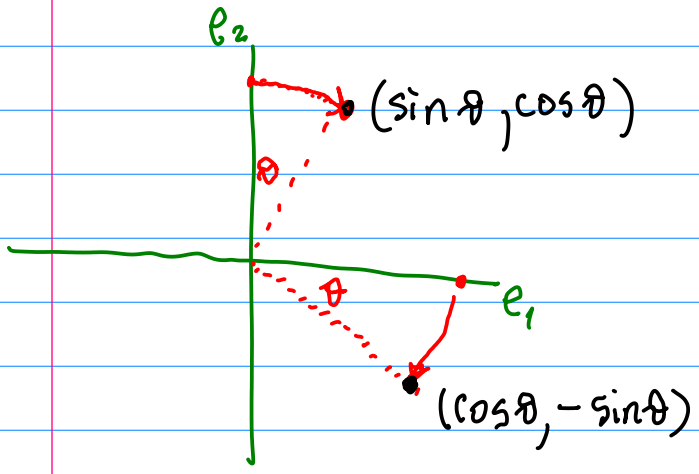
Rotations: Let R_θ be the counterclockwise rotation



$$\begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} = \overbrace{\begin{bmatrix} \cos\theta & ? \\ -\sin\theta & ? \end{bmatrix}}^{R_\theta} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

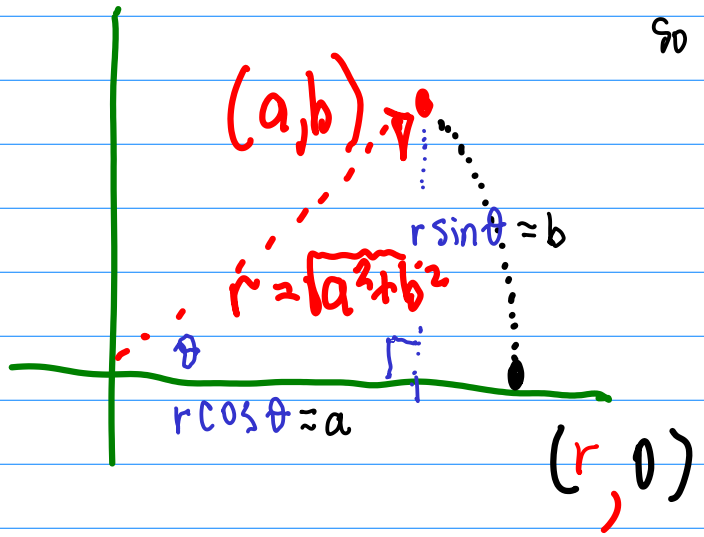
$$\begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

in general $R_\theta = \begin{bmatrix} R_\theta e_1 & \vdots & R_\theta e_2 \\ & \vdots & \end{bmatrix}$

Shorthand notation: $c = \cos\theta$ and $s = \sin\theta$.

Want to use rotation to make thing triangular..

$$\begin{bmatrix} ? & ? \\ 0 & ? \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$



so $\sin\theta = \frac{b}{r}$

$\cos\theta = \frac{a}{r}$

$$r_1' = cr_1 + sr_2$$
$$r_2' = -sr_1 + cr_2$$

$$\begin{matrix} r_1' \\ r_2' \end{matrix} \begin{bmatrix} ac+bs & ec+fs \\ \underbrace{-as+bc}_{0} & -es+fc \end{bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} a & e \\ b & f \end{bmatrix} \begin{matrix} r_1 \\ r_2 \end{matrix}$$

We know any invertible matrix can be written as a product of elementary row operations

- elimination step $\quad r_i \leftarrow r_i - \alpha r_j \quad i \neq j$
- row swap $\quad r_i \leftrightarrow r_j \quad i \neq j$
- scaling step $\quad r_i \leftarrow \beta r_i \quad \beta \neq 0$

$$R_B = E_4 E_3 E_2 E_1 = \left[r_2 \leftarrow r_2 - \tfrac{s}{c} r_1\right]\left[r_2 \leftarrow \tfrac{1}{c} r_2\right]\left[r_1 \leftarrow r_1 + s r_2\right]\left[r_1 \leftarrow c r_1\right]$$

each matrix corresponds to an elementary row operation.

I know this transform can be written as elementary row operations ...

$$r_1' = cr_1 + sr_2$$

$$cr_1 = r_1' - sr_2$$

$$r_1 = \frac{r_1' - sr_2}{c}$$

$$r_2' = -sr_1 + cr_2$$

$$= -s\frac{r_1' - sr_2}{c} + cr_2 = -\frac{s}{c} r_1' + \left(\frac{s^2}{c} + c\right) r_2 = -\frac{s}{c} r_1' + \frac{1}{c} r_2$$

$s^2 + c^2 = 1$

$$\begin{cases} r_1 \leftarrow c r_1 \\ r_1 \leftarrow r_1 + s r_2 \end{cases}$$

$$\begin{cases} r_2 \leftarrow \frac{1}{c} r_2 \\ r_2 \leftarrow r_2 - \frac{s}{c} r_1 \end{cases}$$

```
julia> using LinearAlgebra

julia> using SparseArrays
```

r_p r_q $\cos\theta$ $\sin\theta$

```
function R(n,p,q,c,s)
```

size of matrix

```
julia> function R(n,p,q,c,s)
           M=sparse(Float64,I,n,n)
           M[p,p]=c;  M[p,q]=s
           M[q,p]=-s; M[q,q]=c
           return M
       end
R (generic function with 1 method)

julia> R(5,1,2,cos(1),sin(1))
5×5 SparseMatrixCSC{Float64, Int64} w
  0.540302  0.841471   .    .    .
 -0.841471  0.540302   .    .    .
   .          .       1.0   .    .
   .          .        .   1.0   .
   .          .        .    .   1.0
```

$\sin\theta = \frac{b}{r}$

$\cos\theta = \frac{a}{r}$

```
julia> A=rand(5,5)
5×5 Matrix{Float64}:
 0.79706   0.603343  0.424403   0.500691  0.652836
 0.872642  0.747324  0.150295   0.825133  0.199665
 0.449159  0.730703  0.328155   0.565211  0.199521
 0.678369  0.886857  0.0737307  0.821042  0.115276
 0.281141  0.839837  0.621765   0.867463  0.411338
```

a=

b=

```
julia> a=A[4,1]; b=A[5,1]
0.28114113683427044

julia> r=sqrt(a^2+b^2)
0.7343191997588645

julia> s=b/r
0.382859575136523

julia> c=a/r
0.9238065521126602
```

```
julia> R45=R(5,4,5,c,s)
5×5 SparseMatrixCSC{Float64, Int64} with 7 stored
 1.0   .    .     .         .
  .   1.0   .     .         .
  .    .   1.0    .         .
  .    .    .    0.923807  0.38286
  .    .    .   -0.38286   0.923807

julia> R45*A
5×5 Matrix{Float64}:
 0.79706   0.603343  0.424403  0.500691  0.652836
 0.872642  0.747324  0.150295  0.825133  0.199665
 0.449159  0.730703  0.328155  0.565211  0.199521
 0.734319  1.14082   0.306162  1.0906    0.263978
 0.0       0.436306  0.546162  0.487024  0.335863
```

```
julia> include("givens.jl")
givens (generic function with 1 method)

julia> b=rand(5)
5-element Vector{Float64}:
 0.22602929251578885
 0.266349058338331 45
 0.47479411728655885
 0.4541063714193013
 0.24496055984819953
```

```
julia> RA,Rb=givens(A,b)
```

```
julia> RA
5×5 Matrix{Float64}:
 1.46211   1.57237      0.575634      1.48678     0.668926
 0.0       0.689892     0.421264      0.653204    0.122894
 0.0       1.5563e-18   0.440003     -0.0207609   0.454487
 0.0       1.47044e-17  3.95886e-18   0.199931   -0.0476591
 0.0      -2.3489e-17   5.69875e-18   0.0        -0.130239

julia> Rb
5-element Vector{Float64}:
  0.6858319751320434
  0.307919829128813
 -0.106703042535865 18
 -0.182037673377049
  0.063050893002378 47
```

```
julia> include("backward.jl")
backward (generic function with 1 m

julia> backward(RA,Rb)
5-element Vector{Float64}:
  0.171446046558904 27
  1.3762081888336366
  0.209144037059503 13
 -1.0259034365838833
 -0.484118406362452 07
```

```
julia> x1=A\b
5-element Vector{Float64}:
  0.171446046558903 16
  1.376208188833637
  0.209144037059501 46
 -1.0259034365838828
 -0.484118406362450 46

julia> x2=backward(RA,Rb)
5-element Vector{Float64}:
  0.171446046558904 27
  1.3762081888336366
  0.209144037059503 13
 -1.0259034365838833
 -0.484118406362452 07

julia> norm(x1-x2)
2.6442193687558188e-15
```