Math 466/666: Homework 1 Version A

This homework is based on steps 1 through 5 in the text *First Steps in Numerical Analysis* by Hosking, Joe, Joyce and Turner. Students are encouraged to work in groups and consult resources outside of the required textbook when doing the homework for this class. Please cite any additional sources you used to complete your work.

1. For each of the following numbers

$$57.82156, \qquad 5.782156, \qquad 0.5782156, \qquad 0.05782156$$

(i) round to three significant digits (3S),

(ii) round to three decimal places (3D).

2. Consider the sum

$$S_n = \sum_{k=1}^{n} \frac{1}{k}.$$

Let S_n^* be the approximation of S_n obtained with a hypothetical computer that uses chopping arithmetic. Is it true that $S_n^* \le S_n$? If so explain why; if not provide a counterexample where $S_n^* > S_n$.

3. Evaluate the following using three-digit decimal normalized floating point arithmetic with rounding:

(i) $4.56 \times 10^2 + 8.17 \times 10^2$

(ii) $4.56 \times 10^2 + 8.17 \times 10^1$

(iii) $4.56 \times 10^2 + 8.17 \times 10^{-1}$

(iv) $4.56 \times 10^2 + 8.17 \times 10^{-2}$

4. Let $x \in \mathbf{R}$ and x^* be the approximation of x obtained by rounding to four signifiant digits. Suppose $x^* = 4.562 \times 10^2$.

(i) Find the smallest interval that contains x.

(ii) Find a bound for the absolute error $e_{\text{abs}} = |x - x^*|$.

5. Estimate the accumulated errors in the results of Question 3 assuming that all values are correct to three significant digits. Use either interval arithmetic or bounds based on the fact that accumuated error is the sum of the propagated and generated errors.

6. Find the Taylor series expansion about $x = 0$ for each of the following functions:

(i) $\sin x$

(ii) $\sqrt{1-x}$

(iii) e^{2x}

For each series determine a general remainder term.

7. Use the remainder term in Question 6(iii) to find the degree n of the Taylor polynomial approximation to e^{2x} that gives 4D accuracy for all x between 0 and 1.

8. Evaluate $p(2.1)$ and $p'(2.1)$, where $p(x) = x^3 - 2x^2 + 2x + 3$, using the technique of nested multiplication.