Math 466/666: Homework 2 Version A

This homework is based on steps 6 through 10 in the text *First Steps in Numerical Analysis* by Hosking, Joe, Joyce and Turner. Students are encouraged to work in groups and consult resources outside of the required textbook when doing the homework for this class. Please cite any additional sources you used to complete your work.

1. Sketch curves to roughly locate the roots of the following equations.

   (i) $2x + \cos x = 0$

   (ii) $x + \ln x = 0$

   (iii) $x(x-2) - e^x = 0$

2. What is the maximum error after $n$ iterations of the bisection method?

3. Each equation in Question 1 has only one root. For each equation use bisection method and a suitable programming language to find the root correct to 4D.

4. Compare the results obtained when the bisection method, the method of false position and the secant method are used with starting values 0 and 1 to solve the equation

$$xe^{-x} = 1 - x.$$

5. Consider the equation $e^x + x = 0$.

   (i) Use the iteration $x_{n+1} = -\exp x_n$ with $x_0 = -1$ to find the solution to 3D.

   (ii) [Extra Credit and Math 666] Write $x = -e^x$ and add $Mx$ to both sides to obtain

$$(M+1)x = Mx - e^x.$$

   Solving for $x$ on the left yields the iteration

$$x_{n+1} = \frac{Mx_n - \exp x_n}{M+1}.$$

   Choose a value for $M$ that leads to a faster converging iteration and demonstrate the convergence numerically.

6. Consider finding $\ln a$ by solving $a = e^x$ for $x$ using Newton-Raphson iterations.

   (i) Derive the Newton-Raphson iteration

$$x_{n+1} = x_n - 1 + a\exp(-x_n).$$

   (ii) What happens with this iteration if $a = -3$ and $x_0 = 1$?

   (iii) [Extra Credit and Math 666] Is it true or false that the iteration always converges for $a > 0$ and $x_0 = 1$? If true explain why; if false provide a counter example.

7. Use the Newton–Raphson method to solve to 4D each equation in Question 1.