The matnorm2 subroutine developed in class can be modified such that it iterates until about 15 digits of accuracy have been found in the approximation for $\|A\|_2$. An example of such a modified subroutine follows:

```c
double matnorm2(int n, double A[n][n]){
    double B[n][n], y[n], yk[n];
    bzero(B, sizeof(double)*n*n);
    for(int k=0; k<n; k++) // $B = A^T A$
        for(int i=0; i<n; i++)
            for(int j=0; j<n; j++)
                B[i][j]+=A[k][i]*A[k][j];
    for(int i=0; i<n; i++){ // Choose $x \in \mathbb{R}^n$ randomly
        y[i]=2.0*random()/RAND_MAX+1.0; // and store $x$ in y for now
    }
    double q=0, qk;
    for(int k=1; k<100*n; k++) // $y_k = B^k x/\|B^{k-1}x\|_2$
        multAx(n, n, B, y, yk);
    qk=vecnorm2(n, yk);
    for(int j=0; j<n; j++) // Overwrite y by $y_k/\|y_k\|_2$
        y[j]=yk[j]/qk;
    if(fabs(qk-q)<5e-15*qk) // Converge to 15 digits where
        return sqrt(qk); // $\|A\|_2 \approx (\|B^k x\|_2/\|B^{k-1}x\|_2)^{1/2}$
    q=qk;
    fprintf(stderr,"matnorm2: Failed to converge!\n");
    return sqrt(qk);
}
```

The goal of this project is to modify the above code to create an invmatnorm2 subroutine which approximates $\|A^{-1}\|_2$ and to then combine these two routines to create a third routine matcond2 which approximates the condition number of $A$. 

```c
```
1. Let $A \in \mathbb{R}^{n \times n}$ and $C = AA^T$. If $A$ is nonsingular does it follow that $C$ must also be nonsingular? If so, explain why; if not, give a counter example.

2. Show that $C^{-1} = (A^{-1})^T A^{-1}$ and conclude that $\|A^{-1}\|_2 = \sqrt{\rho(C^{-1})}$.

3. Consider the matrix $B = A^T A$. Do $B^{-1}$ and $C^{-1}$ have the same eigenvalues? If so, explain why; if not, give a counter example.

4. Modify the code in `matnorm2` by replacing `multAx` with `plussolve` to create a routine `invmatnorm2` which approximates $\|A^{-1}\|_2$. You will also need to add a call to `plufact` somewhere before the main loop. Test your routine using the matrix

$$A = \begin{bmatrix} 7 & 1 & 7 \\ 5 & 7 & 7 \\ 3 & 7 & 8 \end{bmatrix}$$

to show that $\|A^{-1}\|_2 \approx 0.73677$.

Please include full program source code and output with your report.

5. Write a routine `matcond2` that computes the condition number

$$\text{cond}_2(A) = \|A^{-1}\|_2\|A\|_2.$$

Approximate the condition number of the matrix given in the previous problem. Please include source code and output with your report.

6. Find $\|A\|_2$, $\|A^{-1}\|_2$ and $\text{cond}_2(A)$ for the special matrix associated with your netid available for download from the course website.

7. [Extra Credit] For $n = 10, 20, 40, 80, \ldots, 1280$ let $K_n$ be the average condition number with respect to the 2-norm of 100 random $n \times n$ matrices with entries uniformly distributed between $-1$ and 1. Plot $K_n$ versus $n$ and try to deduce a relationship between the size of the random matrices and the average condition number.