Reciprocal Distribution Project

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output.

This question explores why the mantissas of the floating-point numbers which appear in typical computations follow the reciprocal distribution.

1a. Consider the primary-school multiplication table which shows all products for the numbers 1 through 9. Make a bar-chart depicting how many of the 81 products in that table begin with the digit 1, how many with a 2 and so forth up to 9.

b. Write a computer program compute all products of the form

$$d_1 \cdot d_2 \cdot d_3 \qquad \text{where} \qquad d_i \in \{1, 2, \ldots, 9\}$$

and count how many begin with the digit 1, how many with a 2 and so forth up to 9. Also compute the percentages in each category. For reference, the output of your program should look like

# digit	count	percent
1	218	29.90
2	137	18.79
3	94	12.89
4	81	11.11
5	46	6.31
6	43	5.90
7	37	5.08
8	37	5.08
9	36	4.94

Please submit your source code—in any language—for this question.

c. In the text it is suggested that the mantissas of the numbers which appear in numerical calculations follow the reciprocal distribution given by the density

$$r(x) = \frac{1}{x \log b} \qquad \text{for} \qquad x \in [1/b, 1].$$

Set $b = 10$ and calculate

$$p_d = \int_{d/10}^{(d+1)/10} r(x)dx \qquad \text{for} \qquad d \in \{1, 2, \ldots, 9\}.$$

to find the probability a numbers starts with the digit d under this hypothesis.

d. Repeat question (ii) for products of the form

$$\pi_n = d_1 \cdots d_n \qquad \text{for} \qquad n = 4, 5, 6$$

and comment on how the percents computed for different values of n compare with the theoretical probabilities in part (iii).

e. Research the reciprocal distribution. State what references you consulted and then provide a theoretical explanation why the mantissas of the floating-point numbers which appear in typical computations follow the reciprocal distribution.