Initial Value Problem Project

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output.

1a. Solve the initial value problem

$$y' = y \sin x \qquad \text{where} \qquad y(0) = 1$$

exactly by multiplying through by the integrating factor

$$\mu = \exp\left\{ \int -\sin x \, dx \right\} = e^{\cos x}$$

to obtain the equation $(\mu y)' = 0$ and then integrating. Find $y(10)$ exactly.

b. Write a program that uses Euler's method to calculate approximate solutions $\tilde{y}$ for the above system. Use your program to find $\tilde{y}(10)$ using step sizes of $h = 10/n$ where n is $16, 32, 64, 128, \ldots, 1048576$.

c. Let $E = |y(10) - \tilde{y}(10)|$. Graph $\log E$ versus $\log h$ to verify the order of convergence for Euler's method numerically.

d. Use Runge-Kutta methods of order 2, 3 and 4 to calculate $\tilde{y}$. Graph $\log E$ versus $\log h$ to verify the order of convergence for each method.

e. Repeat for the initial value problem

$$y' - y \sin x = 1 - x \sin x \qquad \text{where} \qquad y(0) = 1.$$

Are the results the same?