Three Stage Runge–Kuta Methods Project

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output.

1a. The Shu–Osher TVD Runge–Kutta scheme given by the tableau

$$\begin{array}{c|ccc} 0 & & & \\ 1 & 1 & & \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & \\ \hline & \frac{1}{6} & \frac{1}{6} & \frac{2}{3} \end{array}$$

Find the truncation error and order for this method.

b. Use the Shu–Osher TVD Runge–Kutta scheme to approximate the solution to

$$y' = y^2 \cos(t), \qquad y(0) = 0.8$$

on the interval $[0, 8]$. Graph your solution.

c. The exact solution to this equation is

$$y(t) = \frac{y_0}{1 - y_0 \sin(t)}.$$

Let y_n be the approximation of $y(8)$ obtained by the Shu–Osher TVD Runge–Kutta scheme using n equal steps of size $h = 8/n$. Graph $\log |y_n - y(8)|$ versus $\log h$ to verify the order of convergence found in part 1a numerically.

d. The classical Runge–Kutta scheme and the Nystrom Runge–Kutta schemes are given by

$$\begin{array}{c|ccc} 0 & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ 1 & -1 & 2 & \\ \hline & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{array} \qquad \begin{array}{c|ccc} 0 & & & \\ \frac{2}{3} & \frac{2}{3} & & \\ \frac{2}{3} & 0 & \frac{2}{3} & \\ \hline & \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \end{array}$$

respectively. Let z_n be the approximation of $y(8)$ obtained from the classical RK scheme and w_n be obtained from the Nystrom RK scheme using n equal steps of size $h = 8/n$. Compare $\log|z_n - y(8)|$ and $\log|w_n - y(8)|$ to the values of $\log|y_n - y(8)|$ for $n = 50$ and $n = 100$. Which scheme is preferrable when solving the equation in part 1b?