Numerical Quadrature with Simpson Project

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output.

1a. Write a program that computes integrals using Simpson's method. Test your program with $n = 10$ to verify that you obtain

$$\int_1^5 \frac{1}{1+t^2} dt \approx 0.588037206324922.$$

b. Let $g(t) = t^{p-1}e^{-t/2}$. Find t_0 such that $g(t_0) = \max\{\, g(t) : t \geq 0 \,\}$. Show for $M > t_0$ that

$$E_M = \int_M^\infty t^{p-1}e^{-t}dt = \int_M^\infty g(t)e^{-t/2}dt \leq 2M^{p-1}e^{-M}.$$

Find a value of M that ensures $E_M \leq 5 \times 10^{-16}$ for all $p \in [1, 2]$.

c. Let S_n be the approximation of $\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$ given by computing

$$\int_0^M t^{1/2}e^{-t}dt$$

using Simpson's method with n subdivisions. Compute S_n for $n = 2^k$ with $k = 4, \ldots, 10$. Plot a graph of $\log|\Gamma(3/2) - S_n|$ versus $\log h$. What is the slope of the graph? What should the slope have been?

d. Make the substitution $t = u^5$ in the integral of part 1c. Approximate this transformed integral as R_n using Simpson's method with $n = 2^k$ subdivisions where $k = 5, \ldots, 8$. Plot a graph of $\log|\Gamma(3/2) - R_n|$ versus $\log h$. What is the slope of the graph?

e. Analyse the results of 1c and 1d in terms of the error estimates for Simpson's method. Note that

$$\frac{d^4}{dt^4}\left(t^{1/2}e^{-t}\right)$$

is unbounded on $(0, M)$.