Math 467 Final Version A

This is a take-home final exam. Please work independently. If you have questions about the exam feel free to email me at `ejolson@unr.edu` or meet with me in my office.

1. This question is similar to 3.8 followed by 4.6 from our text.

(i) Derive the three-stage Runge-Kutta method that corresponds to the collocation points

$$c = \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{3}}{4} \\ \frac{1}{2} \\ \frac{1}{2} + \frac{\sqrt{3}}{4} \end{bmatrix}.$$

(ii) What is its order?

(iii) Find $r(z)$ for this method and plot the linear stability domain $\mathcal{D}$.

(iv) Is this method A-stable?

2. Problem 5.6 parts a,b and c.

3. The Lorenz system is a three dimensional autonomous ordinary differential equation of the form

$$y' = f(y)$$

where $y(t)$ is a vector in $\mathbf{R}^3$ and

$$f(y) = \begin{bmatrix} -10y_1 + 10y_2 \\ 28y_1 - y_2 - y_1y_3 \\ y_1y_2 - (8/3)y_3 \end{bmatrix}.$$

Use any numerical method you prefer to compute $y(1)$ given the initial condition

$$y(0) = \begin{bmatrix} 2 \\ 3 \\ 15 \end{bmatrix}.$$

Find each component of the vector $y(1)$ to at least 5 significant digits.

4. Matlab has a builtin fast Fourier transform function called `fft`.

(i) Check how fast it is by computing the time T_N to perform a transform of length $N = 2^n$ for values of n ranging from 8 to as large as your computer can handle. Make each timing three times and take the smallest one for T_N. Note that the first timing is usually the longest because of initialization code that has to be run each time a transform of a different length is used. Plot a graph of T_N versus N. Is T_N closer to $N \log N$ or N^2? Explain.

(ii) Repeat the above where $N = p_i$ and p_i is a sequence of prime numbers. Note that a prime numbers can be found in Maple using the `isprime` command. In this case is T_N closer to $N \log N$ or N^2? Explain.