

```
> restart;
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> # Improved more Efficient Version
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> with(LinearAlgebra):
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```
> m:=5;
```

$m := 5$

```
> A:=Matrix([[0,0],[1/3,1/3]]);
```

$$A := \begin{bmatrix} 0 & 0 \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

```
> b:=Vector([1/4,3/4]);
```

$$b := \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

```
> c:=Vector([0,2/3]);
```

$$c := \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix}$$

```
> n:=Dimension(b);
```

$n := 2$

```
> onesvector:=Vector(n,1);
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```
fxi:=Vector(n);
```

```
for j from 1 to n do
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```
    fxi[j]:=f(t+c[j]*h,xi[j]);
```

```
od;
```

```
xirhs:=y(t)*onesvector+h*Multiply(A,fxi);
```

$$\text{onesvector} := \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$fxi := \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$fxi_1 := f(t, \xi_1)$$

$$fxi_2 := f\left(t + \frac{2}{3}h, \xi_2\right)$$

$$xirhs := \begin{bmatrix} y(t) \\ y(t) + h \left(\frac{1}{3} f(t, \xi_1) + \frac{1}{3} f\left(t + \frac{2}{3} h, \xi_2\right) \right) \end{bmatrix}$$

> **xilhs:=Vector(n, i->xi[i]):**

for j from 1 to m-1

do

xilhs:=subs(xi=xilhs,xirhs);

xilhs:=Map(x->series(x,h,j),xilhs);

xilhs:=Map(x->convert(x,polynomial),xilhs);

od:

xilhs;

$$\left[[y(t)], \left[y(t) + \frac{2}{3} f(t, y(t)) h + \left(\frac{2}{9} D_1(f(t, y(t))) + \frac{2}{9} D_2(f(t, y(t)) f(t, y(t))) \right) h^2 + \left(\frac{2}{27} D_{1,1}(f(t, y(t))) \right. \right. \right. \\ \left. \left. \left. + \frac{4}{27} D_{1,2}(f(t, y(t)) f(t, y(t))) + \frac{2}{27} D_{2,2}(f(t, y(t)) f(t, y(t))^2 \right. \right. \right. \\ \left. \left. \left. + \frac{2}{27} D_2(f(t, y(t)) D_1(f(t, y(t))) + \frac{2}{27} D_2(f(t, y(t))^2 f(t, y(t))) \right) h^3 \right] \right]$$

> **R:=Vector(subs(xi=xilhs,fxi));**

$$R := \left[[f(t, y(t))], \left[f\left(t + \frac{2}{3} h, y(t) + \frac{2}{3} f(t, y(t)) h \right. \right. \right. \\ \left. \left. \left. + \left(\frac{2}{9} D_1(f(t, y(t))) + \frac{2}{9} D_2(f(t, y(t)) f(t, y(t))) \right) h^2 + \left(\frac{2}{27} D_{1,1}(f(t, y(t))) \right. \right. \right. \right. \\ \left. \left. \left. + \frac{4}{27} D_{1,2}(f(t, y(t)) f(t, y(t))) + \frac{2}{27} D_{2,2}(f(t, y(t)) f(t, y(t))^2 + \frac{2}{27} D_2(f(t, y(t)) D_1(f(t, y(t))) \right. \right. \right. \\ \left. \left. \left. + \frac{2}{27} D_2(f(t, y(t))^2 f(t, y(t))) \right) h^3 \right] \right]$$

> **T:=y(t+h)-y(t)-h*Multiply(Transpose(b),R);**

$$T := y(t+h) - y(t) - h \left(\frac{1}{4} f(t, y(t)) + \frac{3}{4} f\left(t + \frac{2}{3} h, y(t) + \frac{2}{3} f(t, y(t)) h \right. \right. \\ \left. \left. + \left(\frac{2}{9} D_1(f(t, y(t))) + \frac{2}{9} D_2(f(t, y(t)) f(t, y(t))) \right) h^2 + \left(\frac{2}{27} D_{1,1}(f(t, y(t))) \right. \right. \right. \\ \left. \left. \left. + \frac{4}{27} D_{1,2}(f(t, y(t)) f(t, y(t))) + \frac{2}{27} D_{2,2}(f(t, y(t)) f(t, y(t))^2 + \frac{2}{27} D_2(f(t, y(t)) D_1(f(t, y(t))) \right) \right)$$

> **simplify(subs(seq(eq[i], i=1..m-1), S));**

$$\left(\begin{aligned} & \frac{1}{216} D_{1,1,1}(f(t, y(t))) + \frac{1}{72} D_{1,1,2}(f(t, y(t))) f(t, y(t)) + \frac{1}{72} D_{1,2,2}(f(t, y(t))) f(t, y(t))^2 \\ & + \frac{1}{72} D_{1,2}(f(t, y(t))) D_1(f(t, y(t))) - \frac{1}{72} D_{1,2}(f(t, y(t))) D_2(f(t, y(t))) f(t, y(t)) \\ & + \frac{1}{216} f(t, y(t))^3 D_{2,2,2}(f(t, y(t))) + \frac{1}{72} f(t, y(t)) D_{2,2}(f(t, y(t))) D_1(f(t, y(t))) \\ & - \frac{1}{72} D_2(f(t, y(t))) D_{1,1}(f(t, y(t))) - \frac{1}{72} D_2(f(t, y(t)))^2 D_1(f(t, y(t))) \\ & - \frac{1}{72} D_2(f(t, y(t)))^3 f(t, y(t)) \end{aligned} \right) h^4 + O(h^5)$$