

## Initial Value Problems

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output. If you choose to work in a group of two, please turn in independently prepared reports.

- 1a. Solve the initial value problem

$$y' = y \sin x \quad \text{where} \quad y(0) = 1$$

exactly by multiplying through by the integrating factor

$$\mu = \exp \left\{ \int -\sin x \, dx \right\} = e^{\cos x}$$

to obtain the equation  $(\mu y)' = 0$  and then integrating. Find  $y(10)$  exactly.

- b. Write a program that uses Euler's method to calculate approximate solutions for the above system. Use your program to find  $y_n$  using step sizes of  $h = 10/n$  where  $n$  is 16, 32, 64, 128, ..., 1048576.
- c. Let  $E = |y_n - y(10)|$ . Graph  $\log E$  versus  $\log h$  to verify the order of convergence for Euler's method numerically.
- d. Use Runge-Kutta methods of order 2 and 4 to calculate  $y_n$ . Graph  $\log E$  versus  $\log h$  to verify the order of convergence for each method.
- e. [Extra Credit and Math/CS 667] Repeat for the initial value problem

$$y' - y \sin x = 1 - x \sin x \quad \text{where} \quad y(0) = 1.$$

Are the results the same?