Math/CS 467/667 Take Home Midterm

INSTRUCTIONS: This is a take-home exam. Do not work in groups. You may contact or ask your instructor for help and clarification. Do not ask other persons for help. Part (iii) of each question is extra credit for undergraduates; graduate students must answer part (iii) from 2 questions for full credit.

1. Let B be the matrix given by

1.8635	1.7135	1.9593	1.5685	1.6521
1.7135	1.8984	1.7439	1.6447	1.5253
1.9593	1.7439	2.6919	2.2635	1.6423
1.5685	1.6447	2.2635	2.2056	1.2430
1.6521	1.5253	1.6423	1.2430	1.5505

(i) Use the power method to find the largest eigenvalue.
(ii) Use the inverse power method to find the smallest eigenvalue.
(iii) [$*$] Use the shifted inverse power method to find one more eigenvalue.

2. Consider the two-point linear boundary problem

$$\begin{cases} y'' + \cos(x)y' - (x^2+1)y = 2 \\ y(-2) = y(2) = -1. \end{cases}$$

(i) Solve the above boundary problem for grid sizes of $h = 4/n$ where $n = 4, 8, 16, 32, ..., 256$. Plot your solutions.
(ii) Calculate the value of $y(1)$ to 5 significant digits.
(iii) [$*$] Numerically verify the order of convergence of your solution with theoretical expectations.

3. Consider the initial value problem

$$\begin{cases} y' = \cos(y) + \cos(x) \\ y(0) = 0. \end{cases}$$

(i) Construct a 3rd-order Taylor method for solving this problem.
(ii) Compute $y(10)$ to 5 significant digits using your 3rd-order Taylor method.
(iii) [$*$] Numerically verify the order of convergence of your code to be 3rd order.

4. Consider the general initial value problem

$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0. \end{cases}$$

(i) Starting with Euler's explicit method, use Richardson extrapolation with step sizes $2h$ and $3h$ to create a method that is 2nd-order convergent.
(ii) Starting with Euler's explicit method, use Richardson extrapolation with step sizes h, $2h$ and $3h$ to create a method that is 3rd-order convergent.
(iii) [$*$] Use the 3rd-order method created above to compute $y(2)$ to 5 significant digits when $f(x,y) = -xy + (4x/y)$, $x_0 = 0$ and $y_0 = 1$.