$$\tau_n = y(t_{n+1}) - y(t_n) - \frac{\overbrace{f(t_n, y(t_n))}^{y'(t_n)} + \overbrace{f(t_{n+1}, y(t_{n+1}))}^{y'(t_{n+1})}}{2} h^*$$

$$y(t_{n+1}) = y(t+h) = y(t) + h y'(t) + \frac{h^2}{2} y''(t) + O(h^3)$$

$$y'(t_{n+1}) = y'(t+h) = y'(t) + h y''(t) + O(h^2)$$

$$\tau_n = y(t_n) + h y'(t_n) + \frac{h^2}{2} y''(t_n) + O(h^3) - y(t_n) - \frac{y'(t_n)}{2} h$$

$$- \frac{y'(t_n) + h y''(t_n) + O(h^2)}{2} h = O(h^3)$$

Note if at each step an error of $O(h^3)$ is made and there are N steps of size $h = \frac{(T - t_0)}{N}$, or $N = \frac{T - t_0}{h}$

Then the error after all the steps is "intuitively"

$$E = O(h^3) \cdot N = O(h^{\cancel{3}\,2}) \frac{T - t_0}{\cancel{h}} = O(h^2)$$

Thus the trapezoid method is second order.

This means, PROVIDED IT CONVERGES which we still have to check, that the error $\tilde{E} \leq c h^2$ as $h \to 0$ where c is some constant.