0				
$\frac{1}{2}$	$\frac{1}{2}$			
$\frac{1}{2}$	0	$\frac{1}{2}$		
1	0	0	1	
	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

$\xi_1 = y_n$ $\quad k_1 = f(t_n, y_n)$

$\xi_2 = y_n + \frac{1}{2} h f(t_n, \xi_1)$

$\xi_3 = y_n + \frac{1}{2} h f(t_n + \frac{1}{2} h, \xi_2)$ $\quad k_2 = f(t_n + \frac{1}{2}h, y_n + \frac{1}{2} h k_1)$

$\xi_4 = y_n + h f(t_n + \frac{1}{2} h, \xi_3)$ $\quad k_3 = f(t_n + \frac{1}{2}h, y_n + \frac{1}{2} h k_2)$

$k_4 = f(t_n + h, y_n + h k_3)$

$$y_{n+1} = y_n + h\left(\frac{1}{6} f(t_n, \xi_1) + \frac{1}{3} f(t_n + \frac{1}{2}h, \xi_2) + \frac{1}{3} f(t_n + \frac{1}{2}h, \xi_3) + \frac{1}{6} f(t_n + h, \xi_4)\right)$$

$$y_{n+1} = y_n + h\left(\frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4\right)$$

Use this to analyse the method...
autonomous equations $y' = f(y)$,

let β_i be the value of k_i after substituting the exact solution...

$k_1 = f(y_n)$ $\qquad \beta_1(t) = f(y(t))$

$k_2 = f(y_n + \frac{1}{2} h k_1)$ $\qquad \beta_2(t) = f(y(t) + \frac{1}{2} h \beta_1(t))$

$k_3 = f(y_n + \frac{1}{2} h k_2)$ $\qquad \beta_3(t) = f(y(t) + \frac{1}{2} h \beta_2(t))$

$k_4 = f(y_n + h k_3)$ $\qquad \beta_4(t) = f(y(t) + h\beta_3(t))$

$$y_{n+1} = y_n + h\left(\frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4\right)$$

Compute truncation error... plug in the exact solution and find the residual error

$$\tau_n = y(t_{n+1}) - y(t_n) - h\left(\tfrac{1}{6}\beta_1(t_n) + \tfrac{1}{3}\beta_2(t_n) + \tfrac{1}{3}\beta_3(t_n) + \tfrac{1}{6}\beta_4(t_n)\right)$$

Taylor series expand and cancel to see whats left... (expand around $t = t_n$).

$$y(t_{n+1}) = y(t_n) + h\,y'(t_n) + \frac{h^2}{2}y''(t_n) + \cdots$$

$$= y(t_n) + h\,f(y(t_n)) + \underbrace{\frac{h^2}{2} f_y(y(t_n))\,f(y(t_n))}_{} + \cdots$$

one more term would have a f_{yy} to it...

$$\begin{cases} y' = f(y) \\ y'' = f_y(y)\,y' = f_y(y)\,f(y) \end{cases}$$

$$\beta_2(t_n) = f\left(y(t_n) + \tfrac{1}{2}h\beta_1(t_n)\right)$$

$$= f(y(t_n)) + \tfrac{1}{2}h\beta_1(t_n)\,f_y(y(t_n))$$

$$+ \frac{\left(\tfrac{1}{2}h\beta_1(t_n)\right)^2}{2} f_{yy}(y(t_n)) + \cdots$$