1. Answer one of the following two questions:

(i) Consider the trapezoid formula

\[ T(\alpha, \beta, f) = \frac{f(\alpha) + f(\beta)}{2}(\beta - \alpha). \]

and the resulting quadrature method given by

\[ \int_a^b f(x)dx \approx \sum_{j=0}^{N-1} T(x_j, x_{j+1}, f) \]

where \( N \in \mathbb{N} \) and \( x_j = a + hj \) with \( h = (b - a)/N \). Write a program to approximate the integral

\[ \int_0^{10} \frac{1}{2 + \sin x} \, dx \]

for \( N = 10 \) that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

\[ y' = \frac{y}{t} - \left( \frac{y}{t} \right)^2, \quad y(1) = 1 \]

Approximate the solution to this differential equation on the interval \([1, 2]\) using the RK4 method with \( h = 1/20 \). Print the approximation of \( y(2) \) and the error \( |y(2) - 2/(1 + \log 2)| \) to 15 digits precision. Include the program and the output of the program in your submission.
2. Answer one of the following two questions:

(i) Consider the 3-point Gaussian formula given by

\[ G_3(\alpha, \beta, f) = \frac{\beta - \alpha}{2} \sum_{k=0}^{2} w_k f\left(\alpha + \frac{\beta - \alpha}{2}(x_k + 1)\right) \]

where

\[ x_0 = -\sqrt{3/5}, \quad x_1 = 0, \quad x_2 = \sqrt{3/5}, \]
\[ w_0 = 5/9, \quad w_1 = 8/9, \quad w_2 = 5/9, \]

and the resulting quadrature method given by

\[ \int_{a}^{b} f(x)dx \approx \sum_{j=0}^{N-1} G_3(x_j, x_{j+1}, f) \]

where \( N \in \mathbb{N} \) and \( x_j = a + hj \) with \( h = (b - a)/N \). Write a program to approximate the integral

\[ \int_{0}^{5} \exp(\sin x) \, dx \]

for \( N = 10 \) that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

\[ y' = \frac{1}{t^2} - \frac{y}{t} - y^2, \quad y(1) = -1. \]

Approximate the solution to this differential equation on the interval \([1, 2]\) using Taylor’s 3rd order method with \( h = 1/30 \). Print the approximation of \( y(2) \) and the error \( |y(2) + 1/2| \) to 15 digits precision. Include the program and the output of the program in your submission.