(a) $\frac{\partial w}{\partial t} + c\frac{\partial w}{\partial x} = e^{2x}$ with $w(x,0) = f(x)$

since the coefficient for the $\frac{\partial}{\partial t}$ term is one, I can choose the parameterization in terms of t.

Let $x = x(t)$. Consider $w(x(t), t)$

$$\frac{d}{dt} w(x(t),t) = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} x'(t)$$

So if $x'(t) = c$ then $\frac{d}{dt} w = e^{2x}$

solve these two ODES

$x'(t) = c \qquad x(t) = ct + x(0)$

Therefore

$$\frac{d}{dt} w(x(t),t) = e^{2x(t)} = e^{2(ct + x(0))}$$

$$\frac{dw}{dt} = e^{2(ct+x(0))}$$

$$w(x(t),t) = \frac{1}{2c} e^{2(ct+x(0))} + C_2$$

$$w(x(0),0) = \frac{1}{2c} e^{2x_0} + C_2$$

$$C_2 = w(x(0),0) - \frac{1}{2c} e^{2x(0)}$$

Therefore

$$w(x(t),t) = \frac{1}{2c} e^{2(ct+x(0))} + \underset{f(x(0))}{w(x(0),0)} - \frac{1}{2c} e^{2x(0)}$$

$$w(ct+x(0),t) = \frac{1}{2c}e^{2(ct+x(0))} + f(x(0)) - \frac{1}{2c}e^{2x(0)}$$

$x = ct + x(0)$ so $x(0) = x - ct$

The solution

$$\boxed{w(x,t) = \frac{1}{2c}e^{2x} + f(x-ct) - \frac{1}{2c}e^{2(x-ct)}}$$

to this problem...

(a) $\dfrac{\partial w}{\partial t} + c\dfrac{\partial w}{\partial x} = e^{2x}$ with $w(x,0) = f(x)$