1. Given the lemmata

Lemma 1. The matrix $I - vv^*$ is unitary if and only if $||v||_2^2 = 2$ or v = 0.

Lemma 2. Let x and y be two vectors such that $||x||_2 = ||y||_2$ and $\langle x, y \rangle$ is real. Then there exists a unitary matrix U of the form $I - vv^*$ such that Ux = y

Prove every square matrix is unitarily similar to a triangular matrix.

Answer to question 1 continued (if necessary).

Describe each of the following algorithms and state what the algorithm is used for:

 (i) Newton's method.

(ii) Horners's method.

(iii) The shifted QR algorithm of Francis.

3. Define quadratic order of convergence. Explain mathematically why it is sometimes said that quadratic order of convergence doubles the number of significant digits at each iteration.

4. Define unitary and orthogonal matrices and explain the relationship between them.

5. Define both absolute and relative error. The rounding used by the floating point hardware in a typical computer holds which type of error more or less constant?

6. Explain what is meant by a subordinate matrix norm.

7. Let A ∈ C^{n×n} where n ∈ N. Give formulae for the following matrix norms:
(i) ||A||_∞

(ii) $||A||_1$

8. Let $f: \mathbf{R} \to \mathbf{R}$ be differentiable. The condition number of f is

(A)
$$\frac{xf'(x)}{f(x)}$$

(B)
$$\frac{xf(x)}{f'(x)}$$

(C)
$$\frac{f'(x)}{xf(x)}$$

(D)
$$\frac{f(x)}{xf'(x)}$$

(E) none of these.