MATH 713 MIDTERM REVIEW SHEET VERSION A

The midterm will cover Sections 1.1 through 3.4 from McDonald and Weiss. In addition know and understand the Bernstein polynomials and proof of the Weierstrass theorem on pages 288 through 293 of Davidson and Donsig. The midterm is comprehensive and covers the material from Quiz 1 and the Quiz 1 Review Sheet as well as all homework problems assigned since the beginning of the semester.

Definitions

1. Two definitions of the Cantor set: $\bigcap_{n=1}^{\infty} P_n$ from page 75 and using base 3 expansions in Proposition 2.17 on page 77
2. Two definitions Lebesgue outer measure: λ^* on page 106 and λ^*_δ as given in Lemma 3.9 on page 111
3. Five definitions of the Borel measureable functions
   a. $\hat{C}$ on page 94
   b. $\mathcal{F}$ in Lemma 3.4 on page 97
   c. part (a) and (b) of Lemma 3.5 on page 98
   d. exercise 3.5 which is $\mathcal{F}_1$ in the lecture notes
4. Two definitions of Borel measurable sets: $\mathcal{B}$ on page 95 and $\mathcal{A}(\tau)$ from Theorem 3.4 on page 100
5. Definition of Lebesgue measurable sets $\mathcal{M}$ on page 119
6. Construction of the unmeasurable set S in Lemma 3.12 on page 116

Proofs of Theorems

7. Existence of disjoint equivalence classes Exercise 1.34 on page 25
8. A monotone function f onto $\mathbf{R}$ is continuous as in Proposition 2.20 on page 79
9. $\lambda^*(P) = 0$ as in Proposition 3.6 on page 124 along with the computation on page 75 which shows $\lambda^*(G) = 1$
10. P is uncountable as in Proposition 2.18 on page 77
11. Theorem 3.1 part (a) on page 95
12. Lemma 3.7 on page 99
13. Lemma 3.8 on page 111
14. Theorem 3.8 on page 112
15. $\mathcal{M}$ is an algebra as in the first part of the proof of Theorem 3.11 on page 120
16. $\lambda^*(A \cup B) = \lambda^*(A) + \lambda^*(B)$ for $A, B \in \mathcal{M}$ as in Theorem 3.12 on page 122
17. Proposition 3.4 and 3.5 on page 123

Facts without Proof

18. $\tau \subseteq \mathcal{B} \subseteq \mathcal{M} \subseteq \mathcal{P}(R)$ and $\tau \neq \mathcal{B} \neq \mathcal{M} \neq \mathcal{P}(\mathbf{R})$
19. $\mathcal{B} = \mathcal{A}(\tau)$
20. $\hat{C} = \{\, f\colon \mathbf{R} \to \mathbf{R}$ such that $f^{-1}(B) \in \mathcal{B}$ for all $B \in \mathcal{B}\,\}$
21. Proposition 3.1 and 3.2 on page 107
22. Theorem 3.10 on page 117 and Theorem 3.11 on page 120