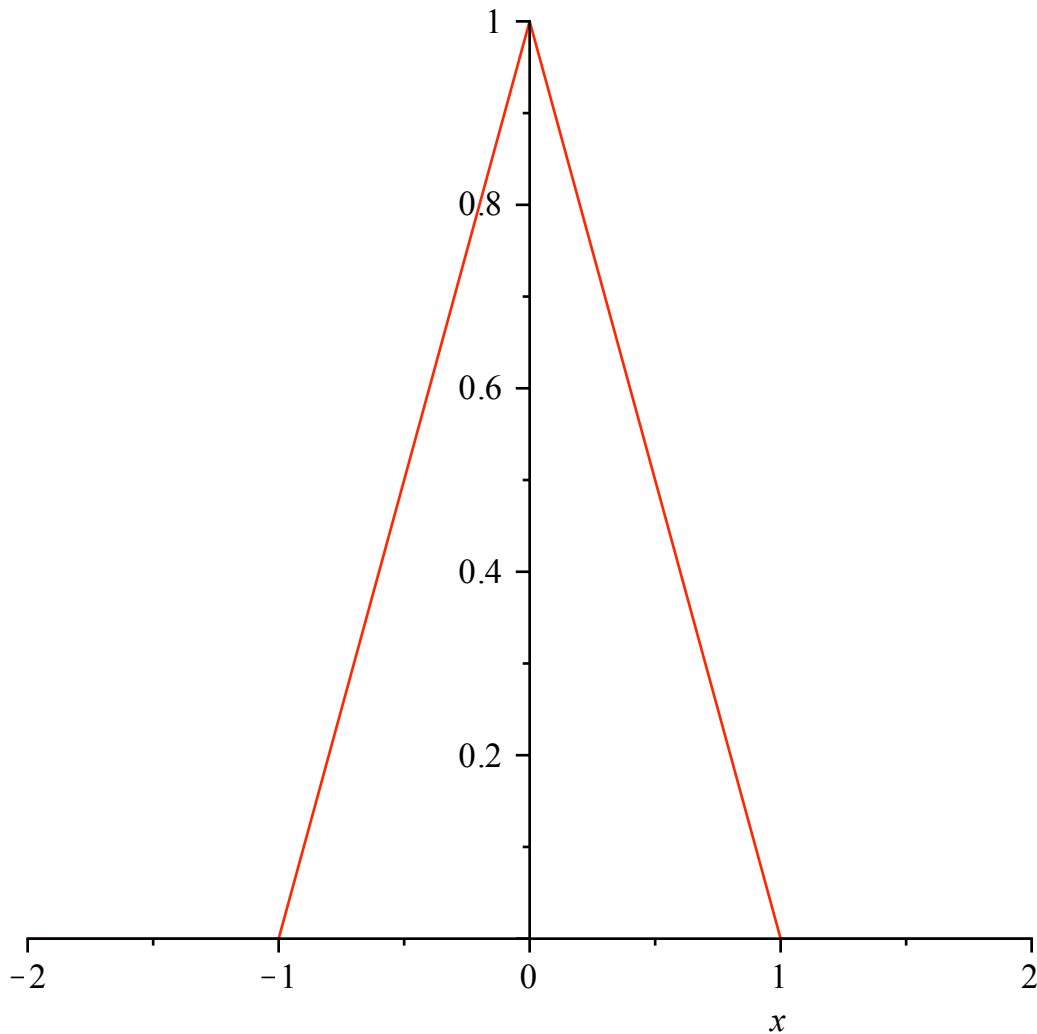


```
> restart;
> phi:=piecewise(x>-1 and x<1, 1-abs(x), 0);
```

$$\phi := \begin{cases} 1 - |x| & -1 < x \text{ and } x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(1)

```
> plot(phi,x=-2..2);
```



```
> phihat:=int(phi*exp(-2*Pi*I*g*x),x=-1..1);
```

$$\text{phihat} := -\frac{1}{4} \frac{(e^{4I\pi g} + 1 - 2e^{2I\pi g}) e^{-2I\pi g}}{\pi^2 g^2}$$

(2)

```
> simplify(phihat);
```

$$-\frac{1}{2} \frac{\cos(2\pi g) - 1}{\pi^2 g^2}$$

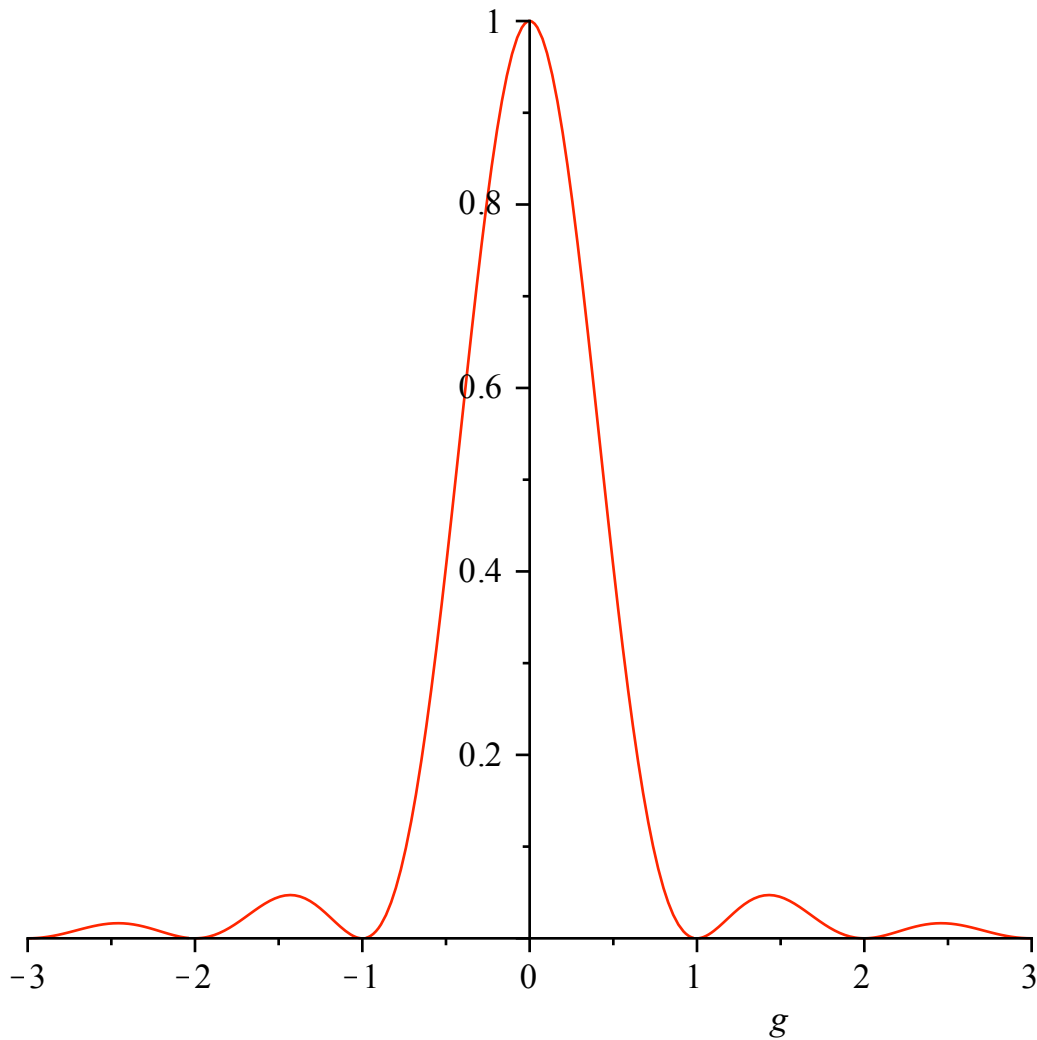
(3)

```
> phihat2:=(sin(Pi*g)/(Pi*g))^2;
```

$$\text{phihat2} := \frac{\sin(\pi g)^2}{\pi^2 g^2}$$

(4)

```
> plot(phihat,g=-3..3);
```



```
> phix:=unapply(phi, x);
      phix := x → piecewise(-1 < x and x < 1, 1 - |x|, 0) (5)
```

```
> Bn:=unapply(int(phix(x)*phix(x-n), x=-infinity..infinity), n);
```

```
Bn := n → piecewise(n ≤ -2, 0, n ≤ -1, -1/3 (n + 1)^3 + 5/3 + 1/2 n ((n + 1)^2 - 1) + n (2 (5)
+ n) + n, n ≤ 0, 1/6 n^3 + 7/3 + 1/2 (2 - n) (n^2 - 1) + n - n^2 + 1/3 (n + 1)^3 + 1/2 (
-2 - n) (n + 1)^2, n ≤ 1, -1/3 (n - 1)^3 - 1/2 (2 - n) (n - 1)^2 - n + 7/3 - 1/6 n^3
- n^2 + 1/2 (-2 - n) (1 - n^2), n ≤ 2, 5/3 + 1/3 (n - 1)^3 + 1/2 n (1 - (n - 1)^2)
- n (2 - n) - n, 2 < n, 0)
```

```
> phid:=sum(Bn(n)*exp(-2*Pi*I*n*g), n=-2..2);
      phid := 2/3 + 1/6 e^{2Iπg} + 1/6 e^{-2Iπg} (7)
```

```
> phid2:=simplify(phid);
```

$$phid2 := \frac{1}{3} \cos(2 \pi g) + \frac{2}{3} \quad (8)$$

> Phi:=1/sqrt(phid2);

$$\Phi := \frac{3}{\sqrt{3 \cos(2 \pi g) + 6}} \quad (9)$$

> phitildehat:=phihat2\*Phi;

$$phitildehat := \frac{3 \sin(\pi g)^2}{\pi^2 g^2 \sqrt{3 \cos(2 \pi g) + 6}} \quad (10)$$

> Cn:=Int(Phi\*exp(2\*Pi\*I\*n\*g),g=0..1);

$$C_n := \int_0^1 \frac{3 e^{2i\pi n g}}{\sqrt{3 \cos(2 \pi g) + 6}} dg \quad (11)$$

> phitilde:=sum(Cn\*phi(x-n),n=-infinity..infinity);

$$phitilde := \sum_{n=-\infty}^{\infty} \left( \int_0^1 \frac{3 e^{2i\pi n g}}{\sqrt{3 \cos(2 \pi g) + 6}} dg \right) \begin{cases} 1 - |x| & -1 < x \text{ and } x < 1 \\ 0 & \text{otherwise} \end{cases} (x - n) \quad (12)$$

> phitildeapp:=simplify(sum(evalf(Re(Cn)\*phix(x-n)),n=-10..10));

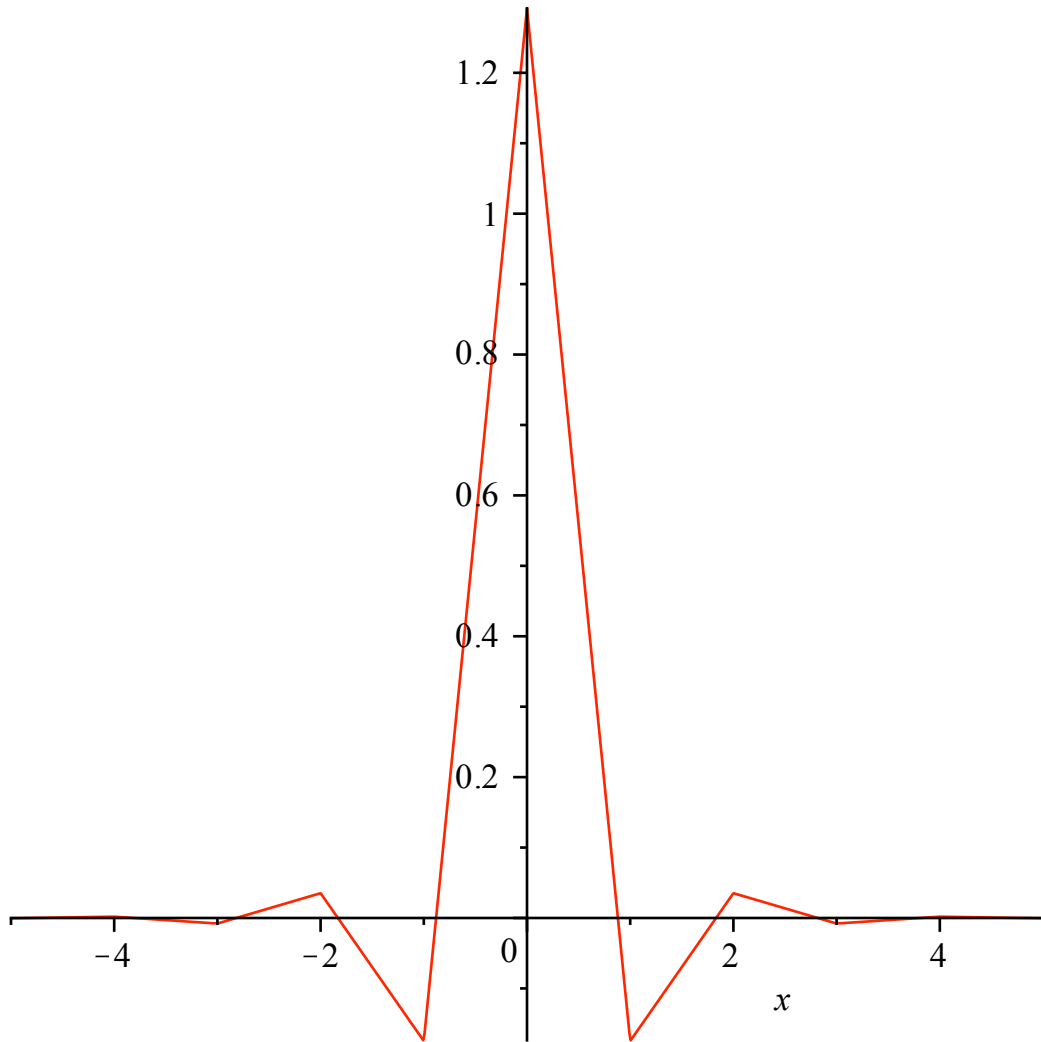
(13)

$phitildeapp :=$

0.	$x \leq -11.$
$0.000004859212417 + 4.417465834 \cdot 10^{-7} x$	$x \leq -10.$
$-0.00002132017436 - 0.000002176192094 x$	$x \leq -9.$
$0.00007555256071 + 0.000008587445135 x$	$x < -8.$
0.000006852999620	$x = -8.$
$-0.0002661543932 - 0.00003412592410 x$	$x \leq -7.$
$0.0009306758707 + 0.0001368498279 x$	$x < -6.$
0.0001095769034	$x = -6.$
$-0.003223412123 - 0.0005554981711 x$	$x \leq -5.$
$0.01102341430 + 0.002293867113 x$	$x < -4.$
0.001847945845	$x = -4.$
$-0.03704153432 - 0.009722370040 x$	$x < -3.$
-0.007874424190	$x = -3.$
$0.1213791871 + 0.04308453708 x$	$x < -2.$
0.03521011289	$x = -2.$
$-0.3845365677 - 0.2098733403 x$	$x \leq -1.$
$1.466338709 x + 1.291675482$	$x \leq 0.$
$1.291675482 - 1.466338709 x$	$x < 1.$
$-0.3845365677 + 0.2098733403 x$	$x < 2.$
0.03521011292	$x = 2.$
$0.1213791871 - 0.04308453708 x$	$x < 3.$
$-0.03704153432 + 0.009722370040 x$	$x < 4.$
0.001847945845	$x = 4.$
$0.01102341430 - 0.002293867113 x$	$x < 5.$
-0.0004459212680	$x = 5.$
$-0.003223412123 + 0.0005554981711 x$	$x < 6.$
0.0001095769034	$x = 6.$
$0.0009306758707 - 0.0001368498279 x$	$x < 7.$
-0.00002727292440	$x = 7.$
$-0.0002661543932 + 0.00003412592410 x$	$x < 8.$
$0.00007555256071 - 0.000008587445135 x$	$x \leq 9.$
$-0.00002132017436 + 0.000002176192094 x$	$x < 10.$
$0.000004859212417 - 4.417465834 \cdot 10^{-7} x$	$x < 11.$
0.	$11. \leq x$

(13)

```
> plot(phitildeapp,x=-5..5);
```



```
> phitildeappx:=unapply(phitildeapp,x);
```

```
phitildeappx := x → piecewise(x ≤ -11., 0., x ≤ -10., 0.000004859212417  
+ 4.417465834 10-7 x, x ≤ -9., -0.00002132017436 - 0.000002176192094 x, x < -8.,  
0.00007555256071 + 0.000008587445135 x, x = -8., 0.000006852999620, x ≤ -7.,  
-0.0002661543932 - 0.00003412592410 x, x < -6., 0.0009306758707  
+ 0.0001368498279 x, x = -6., 0.0001095769034, x ≤ -5., -0.003223412123  
- 0.0005554981711 x, x < -4., 0.01102341430 + 0.002293867113 x, x = -4.,  
0.001847945845, x < -3., -0.03704153432 - 0.009722370040 x, x = -3.,  
-0.007874424190, x < -2., 0.1213791871 + 0.04308453708 x, x = -2., 0.03521011289, x  
≤ -1., -0.3845365677 - 0.2098733403 x, x ≤ 0., 1.466338709 x + 1.291675482, x  
< 1., 1.291675482 - 1.466338709 x, x < 2., -0.3845365677 + 0.2098733403 x, x = 2.,  
0.03521011292, x < 3., 0.1213791871 - 0.04308453708 x, x < 4., -0.03704153432  
+ 0.009722370040 x, x = 4., 0.001847945845, x < 5., 0.01102341430
```

(14)

- 0.002293867113  $x, x = 5.$ , -0.0004459212680,  $x < 6.$ , -0.003223412123  
 + 0.0005554981711  $x, x = 6.$ , 0.0001095769034,  $x < 7.$ , 0.0009306758707  
 - 0.0001368498279  $x, x = 7.$ , -0.00002727292440,  $x < 8.$ , -0.0002661543932  
 + 0.00003412592410  $x, x \leq 9.$ , 0.00007555256071 - 0.000008587445135  $x, x < 10.$ ,  
 -0.00002132017436 + 0.000002176192094  $x, x < 11.$ , 0.000004859212417  
 - 4.417465834  $10^{-7} x, 11. \leq x, 0.)$

> **Hk:=unapply (evalf (Int (phitildeappx (s) \*sqrt (2) \*phitildeappx (2\*s-k) , s=-25..25) ) , k) ;**

$Hk := k \rightarrow \int_{-25.}^{25.} 1.414213562 \text{ piecewise} (s \leq -11., 0., s \leq -10., 0.000004859212417$  (15)  
 + 4.417465834  $10^{-7} s, s \leq -9., -0.00002132017436 - 0.000002176192094 s, s < -8.,$   
 0.00007555256071 + 0.000008587445135  $s, s = -8., 0.000006852999620, s \leq -7.,$   
 -0.0002661543932 - 0.00003412592410  $s, s < -6., 0.0009306758707$   
 + 0.0001368498279  $s, s = -6., 0.0001095769034, s \leq -5., -0.003223412123$   
 - 0.0005554981711  $s, s < -4., 0.01102341430 + 0.002293867113 s, s = -4.,$   
 0.001847945845,  $s < -3., -0.03704153432 - 0.009722370040 s, s = -3.,$   
 -0.007874424190,  $s < -2., 0.1213791871 + 0.04308453708 s, s = -2., 0.03521011289, s$   
 $\leq -1., -0.3845365677 - 0.2098733403 s, s \leq 0., 1.466338709 s + 1.291675482, s$   
 $< 1., 1.291675482 - 1.466338709 s, s < 2., -0.3845365677 + 0.2098733403 s, s = 2.,$   
 0.03521011292,  $s < 3., 0.1213791871 - 0.04308453708 s, s < 4., -0.03704153432$   
 + 0.009722370040  $s, s = 4., 0.001847945845, s < 5., 0.01102341430$   
 - 0.002293867113  $s, s = 5., -0.0004459212680, s < 6., -0.003223412123$   
 + 0.0005554981711  $s, s = 6., 0.0001095769034, s < 7., 0.0009306758707$   
 - 0.0001368498279  $s, s = 7., -0.00002727292440, s < 8., -0.0002661543932$   
 + 0.00003412592410  $s, s \leq 9., 0.00007555256071 - 0.000008587445135 s, s < 10.,$   
 -0.00002132017436 + 0.000002176192094  $s, s < 11., 0.000004859212417$   
 - 4.417465834  $10^{-7} s, 11. \leq s, 0.) \text{ piecewise} (2. s - 1. k \leq -11., 0., 2. s - 1. k \leq -10.,$   
 0.000004859212417 + 8.834931668  $10^{-7} s - 4.417465834 10^{-7} k, 2. s - 1. k \leq -9.,$   
 -0.00002132017436 - 0.000004352384188  $s + 0.000002176192094 k, 2. s - 1. k < -8.,$   
 0.00007555256071 + 0.00001717489027  $s - 0.000008587445135 k, 2. s - 1. k = -8.,$   
 0.000006852999620,  $2. s - 1. k \leq -7., -0.0002661543932 - 0.00006825184820 s$   
 + 0.00003412592410  $k, 2. s - 1. k < -6., 0.0009306758707 + 0.0002736996558 s$   
 - 0.0001368498279  $k, 2. s - 1. k = -6., 0.0001095769034, 2. s - 1. k \leq -5.,$

$-0.003223412123 - 0.001110996342 s + 0.0005554981711 k, 2. s - 1. k < -4.,$   
 $0.01102341430 + 0.004587734226 s - 0.002293867113 k, 2. s - 1. k = -4.,$   
 $0.001847945845, 2. s - 1. k < -3., -0.03704153432 - 0.01944474008 s$   
 $+ 0.009722370040 k, 2. s - 1. k = -3., -0.007874424190, 2. s - 1. k < -2.,$   
 $0.1213791871 + 0.08616907416 s - 0.04308453708 k, 2. s - 1. k = -2., 0.03521011289,$   
 $2. s - 1. k \leq -1., -0.3845365677 - 0.4197466806 s + 0.2098733403 k, 2. s - 1. k \leq 0.,$   
 $2.932677418 s - 1.466338709 k + 1.291675482, 2. s - 1. k < 1., 1.291675482$   
 $- 2.932677418 s + 1.466338709 k, 2. s - 1. k < 2., -0.3845365677 + 0.4197466806 s$   
 $- 0.2098733403 k, 2. s - 1. k = 2., 0.03521011292, 2. s - 1. k < 3., 0.1213791871$   
 $- 0.08616907416 s + 0.04308453708 k, 2. s - 1. k < 4., -0.03704153432$   
 $+ 0.01944474008 s - 0.009722370040 k, 2. s - 1. k = 4., 0.001847945845, 2. s - 1. k$   
 $< 5., 0.01102341430 - 0.004587734226 s + 0.002293867113 k, 2. s - 1. k = 5.,$   
 $-0.0004459212680, 2. s - 1. k < 6., -0.003223412123 + 0.001110996342 s$   
 $- 0.0005554981711 k, 2. s - 1. k = 6., 0.0001095769034, 2. s - 1. k < 7.,$   
 $0.0009306758707 - 0.0002736996558 s + 0.0001368498279 k, 2. s - 1. k = 7.,$   
 $-0.00002727292440, 2. s - 1. k < 8., -0.0002661543932 + 0.00006825184820 s$   
 $- 0.00003412592410 k, 2. s - 1. k \leq 9., 0.00007555256071 - 0.00001717489027 s$   
 $+ 0.000008587445135 k, 2. s - 1. k < 10., -0.00002132017436 + 0.000004352384188 s$   
 $- 0.000002176192094 k, 2. s - 1. k < 11., 0.000004859212417 - 8.834931668 \cdot 10^{-7} s$   
 $+ 4.417465834 \cdot 10^{-7} k, 11. \leq 2. s - 1. k, 0.) ds$

> **Hkf:=k->evalf(Hk(k));**

$Hkf := k \rightarrow \text{evalf}(Hk(k))$

(16)

> **Hkf(2);**

$-0.06910098324$

(17)

> **Gk:=k->(-1)^k\*Hkf(1-k);**

$Gk := k \rightarrow (-1)^k Hkf(1 - k)$

(18)

> **Gka:=[seq(Gk(k),k=-10..10)];**

$Gka := [-0.0001226140533, 0.0002242511073, 0.0005116349364, -0.0009233653788,$

(19)

$-0.002201949941, 0.003883260425, 0.009990594920, -0.01697104673,$

$-0.05194534601, 0.06910098324, 0.3972970872, -0.8176464016, 0.3972970872,$

$0.06910098324, -0.05194534601, -0.01697104673, 0.009990594920, 0.003883260425,$

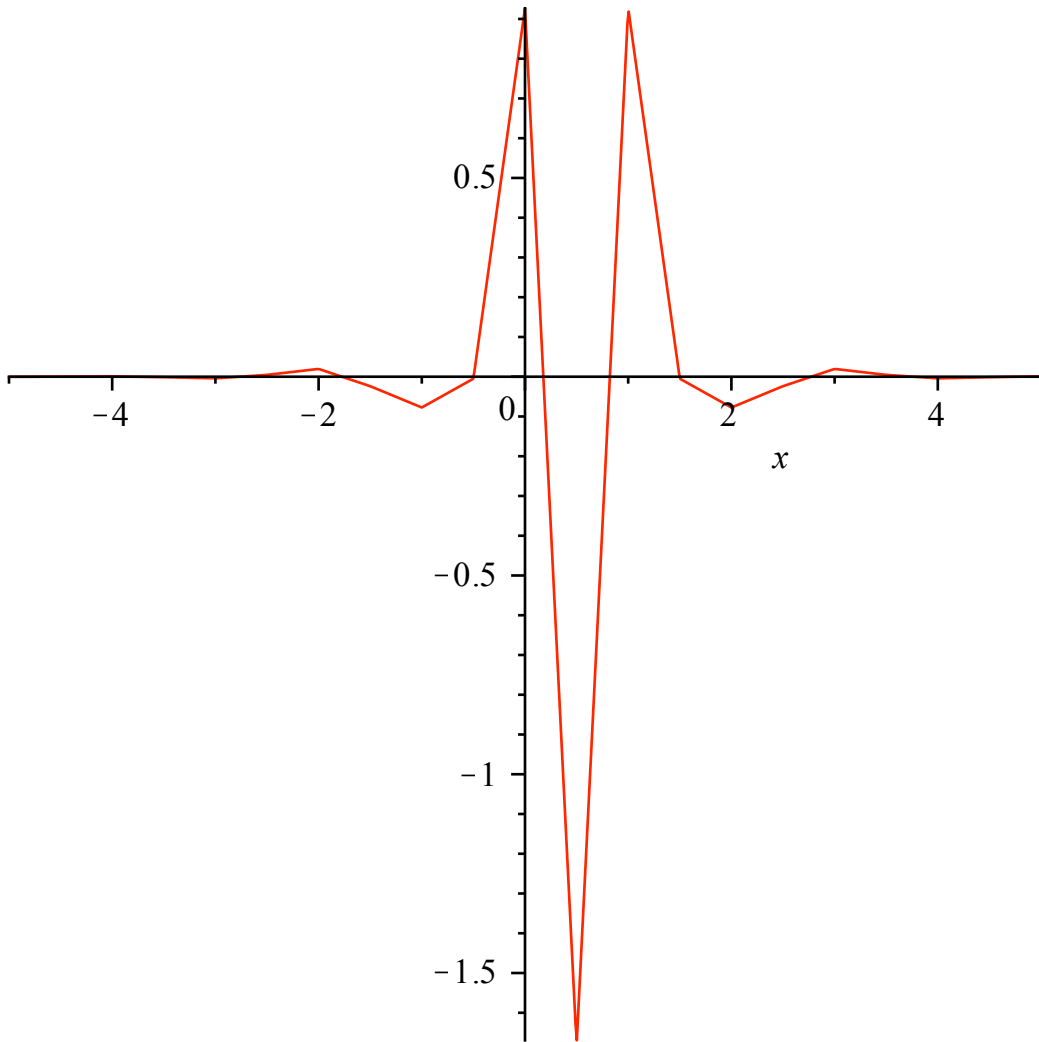
$-0.002201949941, -0.0009233653788, 0.0005116349364]$

> **psi:=simplify(sum(Gka[k+11]\*sqrt(2)\*phitildeappx(2\*x-k),k=-10..10));**

0.	$x \leq -10.50000000$
$-1.608598802 \cdot 10^{-9} - 1.531998860 \cdot 10^{-10} x$	$x \leq -10.$
$1.027244220 \cdot 10^{-8} + 1.034904214 \cdot 10^{-9} x$	$x \leq -9.500000000$
$-3.489171408 \cdot 10^{-8} - 3.719217499 \cdot 10^{-9} x$	$x < -9.$
$-1.418756585 \cdot 10^{-9}$	$x = -9.$
$1.153918130 \cdot 10^{-7} + 1.297895219 \cdot 10^{-8} x$	$x < -8.500000000$
$5.070719500 \cdot 10^{-9}$	$x = -8.500000000$
$-4.517715862 \cdot 10^{-7} - 5.374615360 \cdot 10^{-8} x$	$x \leq -8.$
$0.000001786547463 + 2.260437276 \cdot 10^{-7} x$	$x \leq -7.500000000$
$-0.000006850803616 - 9.256030832 \cdot 10^{-7} x$	$x < -7.$
$-3.715820350 \cdot 10^{-7}$	$x = -7.$
$0.00002685557861 + 0.000003889594379 x$	$x < -6.500000000$
$0.000001573215152$	$x = -6.500000000$
$-0.0001107678557 - 0.00001728324167 x$	$x < -6.$
$-0.000007068405684$	$x = -6.$
$0.0004971998015 + 0.00008404470120 x$	$x \leq -5.500000000$
$-0.0005699844030 x - 0.003099960272$	$x \leq -5.$
$0.005195312134 + 0.001089070077 x$	$x < -4.500000000$
$0.0002944967830$	$x = -4.500000000$
$0.006468533827 + 0.001372008233 x$	$x < -4.$
$0.0009805009000$	$x = -4.$
$-0.01646041367 - 0.004360228644 x$	$x < -3.500000000$
$-0.001199613413$	$x = -3.500000000$
$-0.02169054841 - 0.005854552849 x$	$x < -3.$
$-0.004126889870$	$x = -3.$
$0.04847970844 + 0.01753553276 x$	$x < -2.500000000$
$0.004640876530$	$x = -2.500000000$
$0.07964032468 + 0.02999977925 x$	$x < -2.$
$-0.1540461233 - 0.08684344470 x$	$x < -1.500000000$
$-0.02378095617$	$x = -1.500000000$
$-0.1847589741 - 0.1073186785 x$	$x < -1.$
$-0.07744029550$	$x = -1.$
$0.06753646281 + 0.1449767584 x$	$x < -0.500000000$



```
> plot(psi, x=-5..5);
```



```
> psix:=unapply(psi, x);
```

```
psix := x → piecewise(x ≤ -10.50000000, 0., x ≤ -10., -1.608598802 10-9
- 1.531998860 10-10 x, x ≤ -9.500000000, 1.027244220 10-8 + 1.034904214 10-9 x, x
< -9., -3.489171408 10-8 - 3.719217499 10-9 x, x = -9., -1.418756585 10-9, x <
-8.500000000, 1.153918130 10-7 + 1.297895219 10-8 x, x = -8.500000000,
5.070719500 10-9, x ≤ -8., -4.517715862 10-7 - 5.374615360 10-8 x, x ≤
-7.500000000, 0.000001786547463 + 2.260437276 10-7 x, x < -7., -0.000006850803616
- 9.256030832 10-7 x, x = -7., -3.715820350 10-7, x < -6.500000000,
0.00002685557861 + 0.000003889594379 x, x = -6.500000000, 0.000001573215152, x
< -6., -0.0001107678557 - 0.00001728324167 x, x = -6., -0.000007068405684, x ≤
-5.500000000, 0.0004971998015 + 0.00008404470120 x, x ≤ -5., -0.0005699844030 x
- 0.003099960272, x < -4.500000000, 0.005195312134 + 0.001089070077 x, x =
-4.500000000, 0.0002944967830, x < -4., 0.006468533827 + 0.001372008233 x, x =
```

(21)

$-4., 0.0009805009000, x < -3.500000000, -0.01646041367 - 0.004360228644 x, x =$   
 $-3.500000000, -0.001199613413, x < -3., -0.02169054841 - 0.005854552849 x, x =$   
 $-3., -0.004126889870, x < -2.500000000, 0.04847970844 + 0.01753553276 x, x =$   
 $-2.500000000, 0.004640876530, x < -2., 0.07964032468 + 0.02999977925 x, x <$   
 $-1.500000000, -0.1540461233 - 0.08684344470 x, x = -1.500000000, -0.02378095617,$   
 $x < -1., -0.1847589741 - 0.1073186785 x, x = -1., -0.07744029550, x <$   
 $-0.5000000000, 0.06753646281 + 0.1449767584 x, x = -0.5000000000,$   
 $-0.004951916210, x \leq 0., 0.9271594216 + 1.864222676 x, x < 0.5000000000,$   
 $0.9271594212 - 5.218186122 x, x = 0.5000000000, -1.681933641, x < 1., -4.291026701$   
 $+ 5.218186124 x, x = 1., 0.9271594219, x < 1.500000000, 2.791382104$   
 $- 1.864222683 x, x = 1.500000000, -0.004951918600, x < 2., 0.2125131819$   
 $- 0.1449767338 x, x = 2., -0.07744028540, x < 2.500000000, -0.2920774456$   
 $+ 0.1073185800 x, x < 3., -0.2408906068 + 0.08684384444 x, x = 3., 0.01964092651, x$   
 $< 3.500000000, 0.1096452072 - 0.03000142686 x, x = 3.500000000, 0.004640213110, x$   
 $< 4., 0.06599021005 - 0.01752857055 x, x = 4., -0.004124072120, x < 4.500000000,$   
 $-0.02741948617 + 0.005823853516 x, x < 5., -0.02149944406 + 0.004508288598 x, x$   
 $= 5., 0.001041998928, x < 5.500000000, 0.01291629692 - 0.002374859597 x, x$   
 $= 5.500000000, -0.0001454308660, x < 6., -0.002067843941 + 0.0003495296502 x, x$   
 $= 6., 0.00002933395960, x \leq 6.500000000, 0.0004593365107 - 0.00007166709184 x, x$   
 $< 7., -0.0001109414232 + 0.00001606797490 x, x = 7., 0.000001534401083, x$   
 $< 7.500000000, 0.00002831200578 - 0.000003825372101 x, x = 7.500000000,$   
 $-3.782849680 \cdot 10^{-7}, x < 8., -0.000007406689302 + 9.371205781 \cdot 10^{-7} x, x = 8.,$   
 $9.027532140 \cdot 10^{-8}, x < 8.500000000, 0.000001868367526 - 2.222615258 \cdot 10^{-7} x, x < 9.,$   
 $-4.748114682 \cdot 10^{-7} + 5.340659135 \cdot 10^{-8} x, x \leq 9.500000000, 1.440820865 \cdot 10^{-7}$   
 $- 1.535935919 \cdot 10^{-8} x, x < 10., -4.270949721 \cdot 10^{-8} + 4.302912784 \cdot 10^{-9} x, x$   
 $< 10.500000000, 6.712243202 \cdot 10^{-9} - 6.392612574 \cdot 10^{-10} x, 10.50000000 \leq x, 0.)$

**> int (psi\*psi,x=-10..10) ;**  
0.9999999240 (22)

**> int (phitildeapp\*phitildeapp,x=-10..10) ;**  
1.000000000 (23)

**> int (phitildeappx(x)\*psix(x),x=-10..10) ;**  
2.777365601 10<sup>-8</sup> (24)

**> int (phitildeappx(x)\*phitildeappx(x-1),x=-10..10) ;**  
5.456941586 10<sup>-10</sup> (25)

```
> int(psix(x)*psix(x-1),x=-10..10);  
3.060328658 10-7
```

(26)