M119 Sample Midterm, Fall 2002

Last Name: ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚

First Name: ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚

Instructor: ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚

Directions: This exam consists of 20 multiple choice questions. For each multiple choice question, circle the letter below that corresponds to the **best** choice. **To receive credit**, your answer **must appear on this cover page**.

1) A B C D E
2) A B C D E
3) A B C D E
4) A B C D E
5) A B C D E
6) A B C D E
7) A B C D E
8) A B C D E
9) A B C D E
10) A B C D E
11) A B C D E
12) A B C D E
13) A B C D E
14) A B C D E
15) A B C D E
16) A B C D E
17) A B C D E
18) A B C D E
19) A B C D E
20) A B C D E
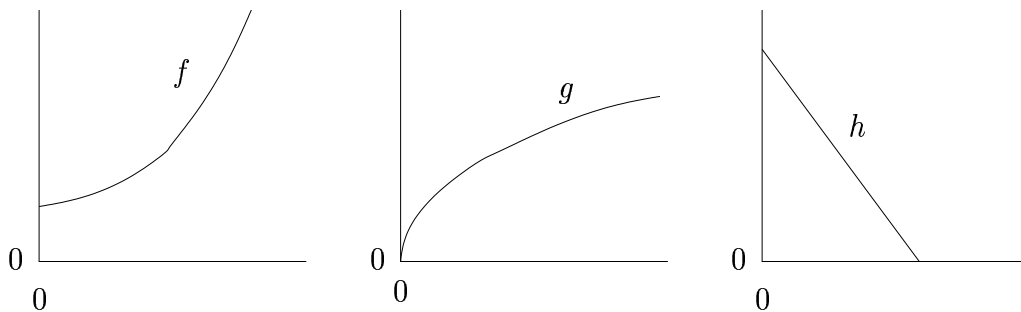
(1) This problem refers to the three graphs above. Which of the following could be true?
  A. f is an increasing function, g is an exponential function, and h is a linear function.
  B. f is concave up, g is a power function, and h is an exponential function.
  C. f is a power function, g is concave down, and h is a power function.
  D. f is an exponential function, g is a power function, and h is decreasing.
  E. None of the above is true.

(2) A thermometer has two scales upon it: Fahrenheit and Celsius. When the thermometer reads x Celcius, it will read $f(x)$ Fahrenheit. Which of the following represents the linear relationship betweeen the Fahrenheit scale and the Celsius scale? (*Recall that water boils at 100 Celsius or 212 Fahrenheit, and water freezes at 0 Celcius or 32 Fahrenheit.*)

  A. $f(x) = 1.8 \cdot x + 32$
  B. $f(x) = 212(x - 100) + 32$
  C. $f(x) = (5/9) \cdot x - 32$
  D. $f(x) = 180(x - 32)$
  E. $f(x) = (112/32) \cdot x - 32$

(3) Suppose that a company that creates widgets has the cost function $C(x) = 2400 + .5 \cdot x$ and the revenue function $R(x) = .75 \cdot x$. Which of the following is true?

  A. At the break-even point, the company produces 9600 widgets.
  B. The profit function is $\Pi(x) = 2400 + .25 \cdot x$.
  C. The fixed costs equal 1200.
  D. The variable cost per unit is .75.
  E. None of the above is true.

(4) A patient drinks a cup of coffee containing about 100 mg of caffeine. Every hour, the amount of caffeine remaining in the patient's body decreases by 17%. In 5 hours, the amount still present is

  A. $100(.17)^5$
  B. $100e^{-.17 \cdot 5}$
  C. $100(1.17)^5$
  D. $100(.83)^5$
  E. None of the above is correct.

The table below gives the percentage of Indiana high school seniors reporting that they had used marijuana in the given year. (Data from the Indiana Prevention Resource Center.)

year of usage	1991	1993	1995	1997	1999
percentage seniors using marijuana	34.2	28.3	34.6	37.1	37.1

This table should be used in the following 2 questions.

(5) The average rate of change between 1993 and 1999 in the percentage of seniors reporting use of marijuana was approximately
A. 1.46
B. 8.80
C. 4.40
D. 2.92
E. 6.30

(6) Let $p(t)$ denote the percentage of seniors using marijuana in year t. Using the table above, we estimate $p''(1997)$ to be

A. positive
B. negative
C. equal to the average rate of change from 1993 to 1997
D. equal to the average rate of change from 1995 to 1997
E. none of the above

(7) Let k be a constant. The derivative of $2e^{k \cdot x^2}$ equals

A. $\frac{4}{k} \cdot e^{kx^2}$.
B. $2kx \cdot e^{2kx}$.
C. $\frac{2}{k} \cdot e^{2kx}$.
D. $4kx \cdot e^{kx^2}$.
E. None of the above.

(8) Let $h(x) = f(g(x))$ where f and g are two differentiable functions satisfying $g(2) = 1$, $g'(2) = 6$, $f(1) = 2$, and $f'(1) = 3$. Then which of the following *must* be true?

A. $h'(1) = 18$.
B. $h(1) = 1$.
C. $h'(2) = 18$.
D. $h(1) = 2$.
E. None of the above is true.

(9) Find the equation of the line that is tangent to $y = 3\ln(2x)$ at the point $(\frac{1}{2}, 0)$.

A. $y = 3(x - \frac{1}{2})$
B. $y = \frac{3}{2x} - 1$
C. $y = 6(x - \frac{1}{2})$
D. $y = \frac{3}{2}(x - 1)$
E. $y = \frac{2}{3}(x + 1)$

(10) The following calculation contains an error. On which line does the error occur?

(A) $\dfrac{d}{dx}\left(\ln(x^2)\cdot e^{7x}\right) = \left(\dfrac{d}{dx}\ln(x^2)\right)\cdot e^{7x} + \ln(x^2)\cdot\cdot\left(\dfrac{d}{dx}e^{7x}\right)$

(B) $= \left(\dfrac{d}{dx}\ln(x^2)\right)\cdot e^{7x} + \ln(x^2)e^{7x}$

(C) $= \left(\dfrac{d}{dx}x^2\right)\cdot\dfrac{1}{x^2}\cdot e^{7x} + \ln(x^2)\cdot e^{7x}$

(D) $= 2x\cdot\dfrac{1}{x^2}\cdot e^{7x} + \ln(x^2)\cdot e^{7x}$

(E) $= \left(2x\cdot\dfrac{1}{x^2} + \ln(x^2)\right)\cdot e^{7x}.$

(11) The following table shows some data for the median price of a house during the years 1970 to 1988.

year	1970	1976	1982	1988
price (in dollars)	22,154	33,247	49,895	74,879

According to this data, the price of a house grew

A. by 7% each year.
B. at a continuous annual rate of 10% each year.
C. by 50% each year
D. at a continuous annual rate of 6% each year.
E. None of the above.

(12) If $f(x) = 4x - 5$, then $2\cdot f(x-2)$ equals

A. $8x - 26$
B. $8x - 21$
C. $8x - 16$
D. $8x - 10$
E. None of the preceding.

(13) Let $J(s) = \dfrac{s^2+1}{s+3}$. Then the derivative, $J'(s)$, equals

A. $\dfrac{-s^2-6s+1}{(s+3)^3}$
B. $\dfrac{3s^2+6s+1}{(s+3)^3}$
C. $\dfrac{s^2+6s-1}{(s+3)^3}$
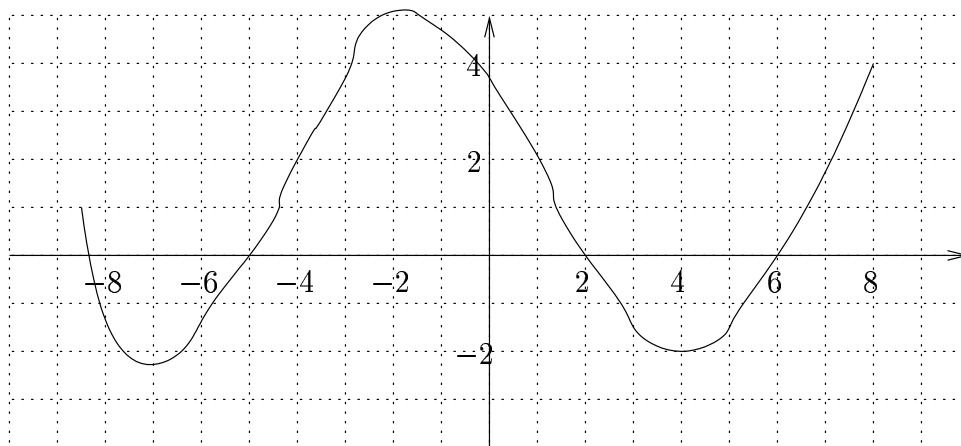D. $\dfrac{-3s^2-6s+1}{(s+3)^3}$
E. None of the preceding.

FIGURE 1. The graph of f in problem 14

(14) Let f be the function whose graph appears in the figure above. This function is increasing for
A. $-8 \leq x \leq -2$ and $-2 \leq x \leq 4$
B. $-7 \leq x \leq -2$ and $4 \leq x \leq 8$
C. $-8 \leq x \leq -7$ and $4 \leq x \leq 8$
D. $-2 \leq x \leq 4$
E. None of the above.

(15) Let f be a function. Which of the following statements about f must be true.

A. If $f'(x)$ is increasing, then $f(x)$ is increasing and $f''(x)$ is positive.
B. If $f(x)$ is concave up, then both $f'(x)$ and $f''(x)$ are positive.
C. If $f'(x)$ is positive, then $f(x)$ is increasing and $f''(x)$ is positive.
D. If $f''(x)$ is positive, then $f'(x)$ is negative and $f(x)$ is concave up.
E. None of the above must be true.

(16) Let $C(q)$ be the cost of producing q Furbies. Suppose that $C(300) = \$2200$ and $C'(300) = 25$. The cost of producing 303 Furbies is approximately:

A. \$2275.
B. \$2125.
C. \$2175.
D. \$2225.
E. None of the above.

(17) A golf ball is hit by a club. The distance traveled distance t seconds after being hit is given by the following table.

t (in seconds)	0.0	0.2	0.4	0.6	0.8
distance (in meters)	0	5	9	14	18

Based on this data, the best estimate of the object's speed 0.6 seconds after being hit is

A. 14.7 meters/second.
B. 22.5 meters/second.
C. 26.8 meters/second.
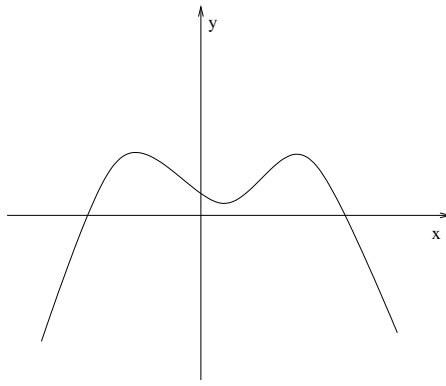D. 31.3 meters/second.
E. None of the above.

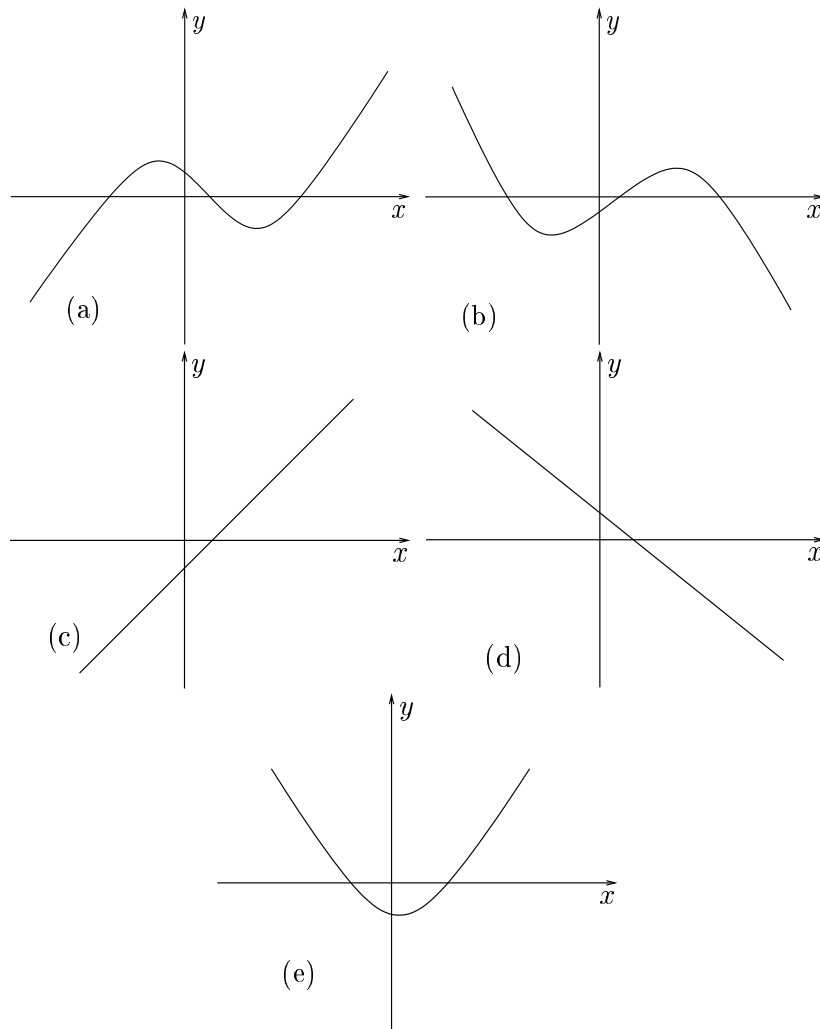FIGURE 2. The graph of f for problem 18

(a)

(b)

(c)

(d)

(e)

FIGURE 3. Choices of f' for problem 18

(18) The graph of a function f appears in the figure at the top of this page. Which of the five graphs sketched above could be the graph of the derivative of f?
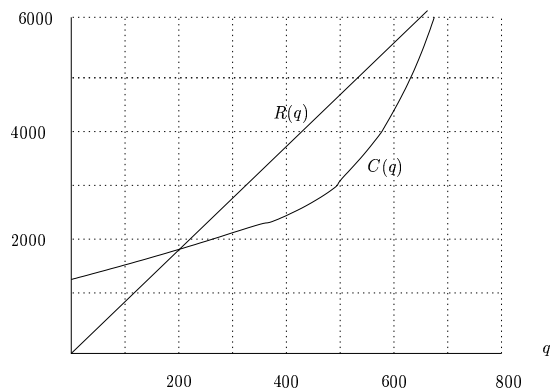
FIGURE 4. The graphs of the revenue and cost functions for problem 19

(19) Above are the graphs of the revenue and cost functions for a certain manufacturer. Assume that the firm's goal is to maximize profits. Which of the following is true?

    A. The firm should make exactly 150 units.
    B. The firm should not make the 400th unit.
    C. The firm should make exactly 700 units.
    D. The firm should not make the 600th unit.
    E. None of the above.

(20) Let $g(t) = (t^3 + 1)^2$. Find the second derivative, $g''(t)$.

    A. $4t^2$
    B. $12t \cdot (t^3 + 1) + 18t^4$
    C. $6t^2 \cdot (t^3 + 1) + 4t^2$
    D. $4t^2 + 1$
    E. None of the above.