Nonlinear Schrödinger Equation

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output. If you choose to work in a group of two, please turn in independently prepared reports.

1. Consider the nonlinear Schrödinger equation
\[
\begin{aligned}
iv_t &= v_{xx} + 2|v|^2v & \text{for } (x, t) \in \mathbb{R} \times (0, T) \\
v(x, 0) &= f(x) & \text{for } x \in \mathbb{R}.
\end{aligned}
\]
Suppose for some \( L > 0 \) that \( f(x) = f(x+L) \) for \( x \in \mathbb{R} \). Prove the resulting solution \( v \) satisfies \( v(x,t) = v(x+L,t) \) for all \((x,t) \in \mathbb{R} \times (0,T)\). We shall say that \( v \) satisfies the nonlinear Schrödinger equation with \( L \)-periodic boundary conditions.

2. Let \( v \) be a solution to the nonlinear Schrödinger equation with \( L \)-periodic boundary conditions. Let \( \Delta x = L/K \) and \( \Delta t = T/N \) and consider the finite difference method for approximating \( v \) on \([0, L] \times [0, T]\) given by
\[
\begin{aligned}
&u_{k+1}^{n+1} = u_{k}^{n-1} - \frac{2i\Delta t}{\Delta x^2} \delta^2 u_{k}^{n} - 4i\Delta t|u_{k}^{n}|^2 u_{k}^{n} & \text{for } n = 1, \ldots, N-1 \\
&\quad \quad \quad \quad \text{and } k = 1, \ldots, K \\
&u_{1}^{1} = u_{k}^0 - \frac{i\Delta t}{\Delta x^2} \delta^2 u_{k}^0 - 2i\Delta t|u_{k}^0|^2 u_{k}^0 & \text{for } k = 1, \ldots, K \\
&u_{0}^{0} = u_{k}^n \quad \text{and } u_{K+1}^{n} = u_{1}^{n} & \text{for } n = 0, \ldots, N \\
&u_{k}^{0} = f(k\Delta x) & \text{for } k = 1, \ldots, K.
\end{aligned}
\]
For \( L = 10 \) and \( T = 0.2 \) choose \( K \) and \( N \) sufficiently large to approximate \( v(5,0.2) \) to at least 3 significant digits for the personalized initial condition \( f \) given below

\[
\begin{align*}
f_{\text{Alexander}}(x) &= 2 \exp(3i \sin(\omega x)) + \sin(3\omega x) \\
f_{\text{Anthony}}(x) &= \exp(i \omega x) - \sin(2\omega x) + \exp(5i \omega x) \\
f_{\text{Brian}}(x) &= 0.5 \exp(i \omega x) + 0.5 \sin(2\omega x) - 2i \cos(3\omega x) \\
f_{\text{Jordan}}(x) &= 2 \exp(-i \omega x) - i \cos(3\omega x) \\
f_{\text{Joseph}}(x) &= 0.5 \exp(i \omega x) - \exp(2i \omega x) + i \exp(3i \omega x) + 0.5 \exp(5i \omega x) \\
f_{\text{Kyle}}(x) &= i \sin(\omega x) + \cos(2\omega x) + \exp(-3i \omega x) \\
f_{\text{Masakazu}}(x) &= 1 + \exp(i \omega x) \\
f_{\text{Sarah}}(x) &= \exp(i \omega x) + \exp(-2i \omega x) + \exp(3i \omega x) \\
f_{\text{Shijie}}(x) &= \sin(3\cos(\omega x)) + 2 \exp(2i \omega x)
\end{align*}
\]
where \( \omega = \pi/5 \).

3. Approximate \( v(5,0.2) \) using one of the other methods found, for example, in


Please include a bibliographic reference to the method you implemented as a comment in your source code.