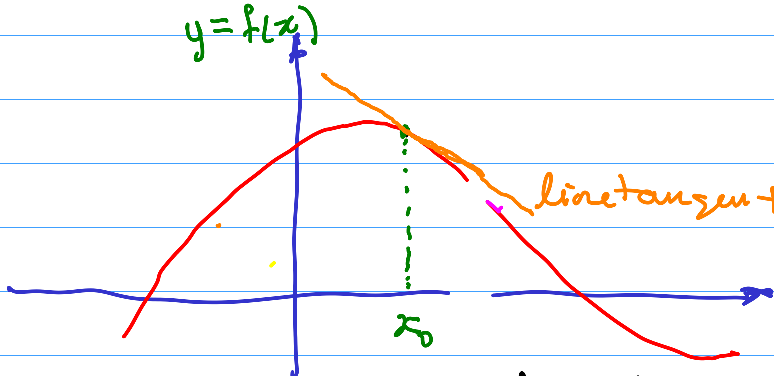
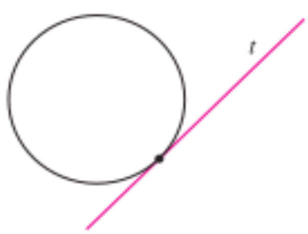
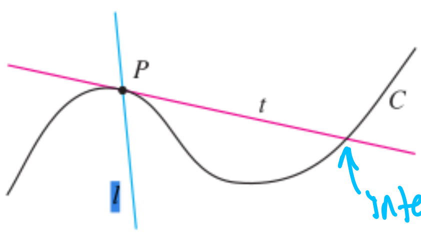
§2.1. The Tangent Problem...



What is that mathematically?



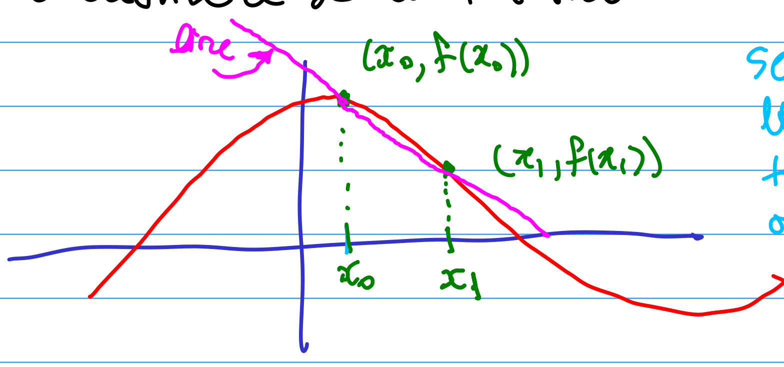
tangent to a circle: Euclid defined this as the line that intersects the circle in only one point.



doesn't work.

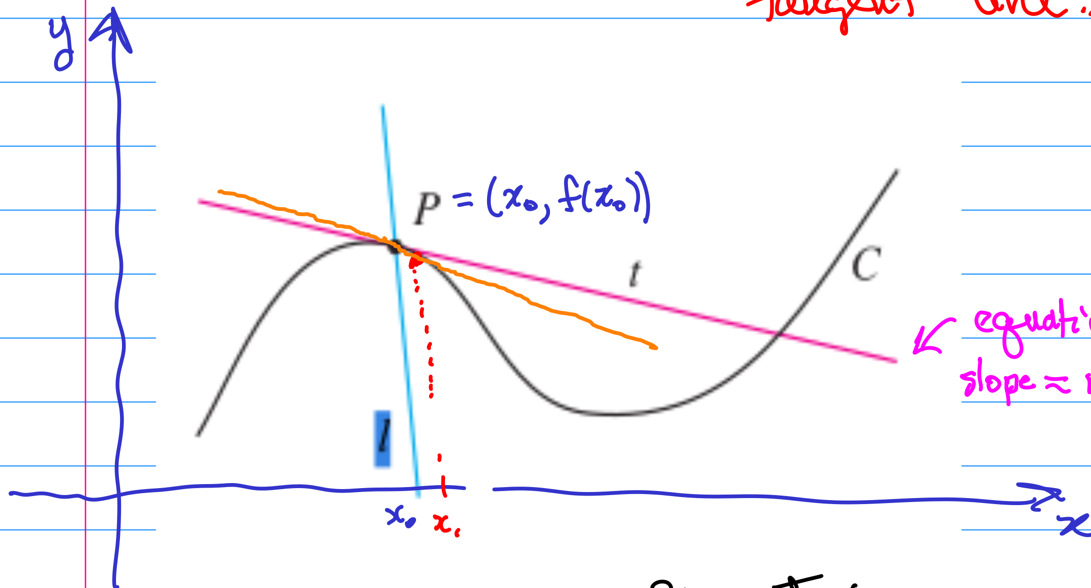
Idea: If I know how to approximate something better and better then in fact I know what that something is mathematically..

Easy to define a secant line



secant is the the line passing through two specified points on the curve as the points x_0 and x_1 get close together the secant line visually approximates the tangent

view this as definition of the tangent line...



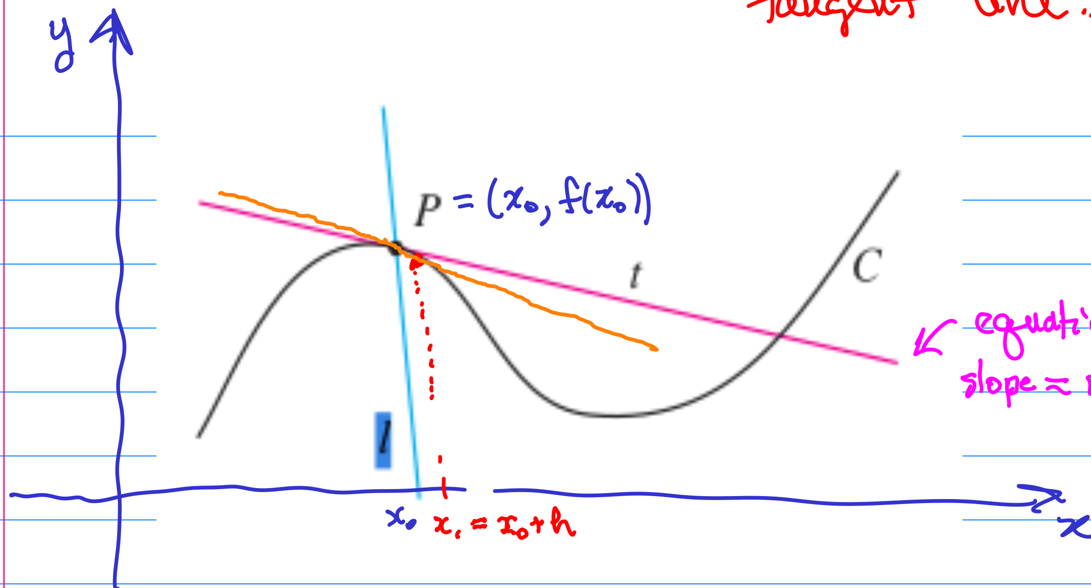
equation for tangent line.
slope = m

Since I know $(x_0, f(x_0))$ is on the line, the only information missing is the slope of the tangent

The slope of the approximating secant line approximates the slope of the tangent...

$$m \approx \text{slope of secant line} = \underbrace{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}_{\text{difference quotient}}$$

tangent line...



If h is small then the distance between x_0 and x_1 is small

$m \approx$ slope of secant line $= \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{f(x_0+h)-f(x_0)}{h}$

$f(x) = \sin(x) \qquad x_0 = 2$

h	Slope of the secant
1	$\dfrac{f(x_0+1)-f(x_0)}{1} = \dfrac{\sin(2+1)-\sin 2}{1} \approx -0.7681\ldots$
.1	$\dfrac{f(x_0+.1)-f(x_0)}{.1} \approx -0.46088\ldots =$
.01	$\dfrac{f(x_0+.01)-f(x_0)}{.01} = \sim 0.4200\ldots =$
.001	$\dfrac{f(x_0+.001)-f(x_0)}{.001} = -0.4166\ldots \approx \cos(2) = -0.41614\ldots$

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julia> Q(h)=(f(x0+h)-f(x0))/h
Q (generic function with 1 method)

julia> f(x)=sin(x)
f (generic function with 1 method)

julia> x0=2
2

julia> Q(1)
-0.7681774187658145

julia> Q(.1)
-0.46088060176808

julia> Q(.01)
-0.42068635004807176

julia> Q(.001)
-0.416601415864859

julia> cos(2)
-0.4161468365471424
```