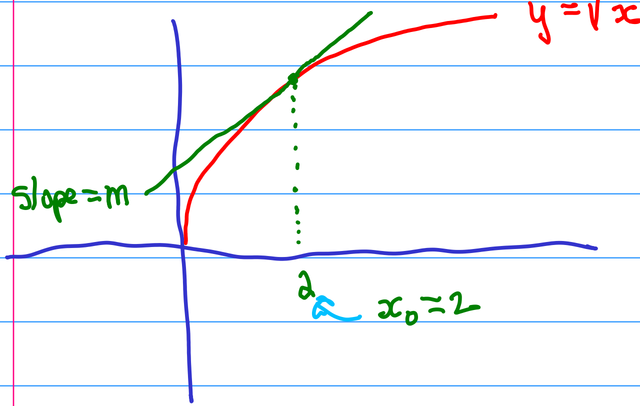
Another derivative example



$f(x) = \sqrt{x}$

limit of approximating secant lines

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

```
julia> Q(h)=(sqrt(2+h)-sqrt(2))/h
Q (generic function with 1 method)

julia> Q(0.1)
0.34924112245848793

julia> Q(0.01)
0.35311255026873045

julia> Q(0.001)
0.3535092074644641
```

h	$\frac{\sqrt{2+h} - \sqrt{2}}{h}$
.1	0.34924112245848793
.01	0.35311255026873045
.001	0.3535092074644641
↓	
0	0.353 . . . . .

Algebra to see what happens when $h \to 0$

make a difference of squares...

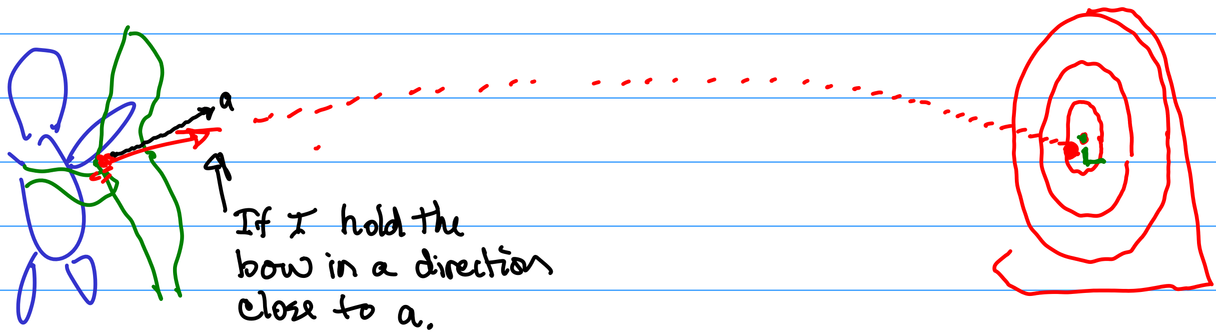
$$\frac{(\sqrt{2+h} - \sqrt{2})}{h} \cdot \frac{(\sqrt{2+h} + \sqrt{2})}{\sqrt{2+h} + \sqrt{2}} = \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})} = \frac{2+h-2}{h(\sqrt{2+h} + \sqrt{2})} = \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$m = \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} = \lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}} = \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}} =$$

```
julia> 1/(2*sqrt(2))
0.35355339059327373
```

actual slope of tangent

> **1 Intuitive Definition of a Limit** Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write
>
> $$\lim_{x \to a} f(x) = L$$
>
> and say "the limit of $f(x)$, as x approaches a, equals L"
>
> if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by restricting x to be sufficiently close to a (on either side of a) but not equal to a.



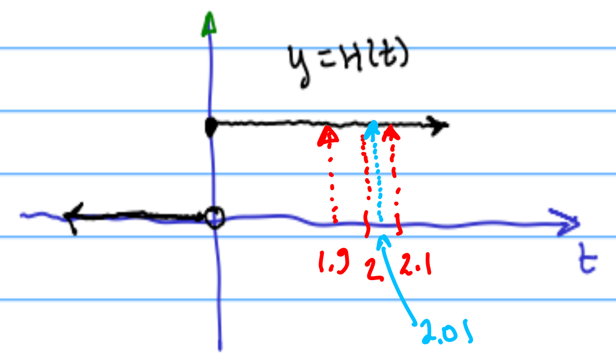

```
julia> Q(0.0)
NaN
```

Cant hold the bow in exactly the direction of a.

- Idea of limit is cant just plug in 0 but there is a approximating process that gets close the the "bullseye" as much as you like...

Example.

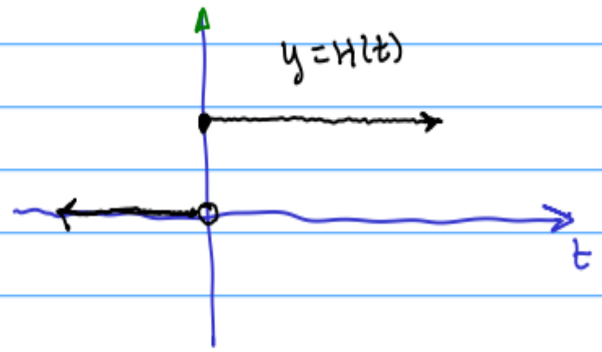
$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$



$$\lim_{t \to 2} H(t) = \lim_{\substack{t \to 2 \\ t \geq 0}} 1 = 1$$

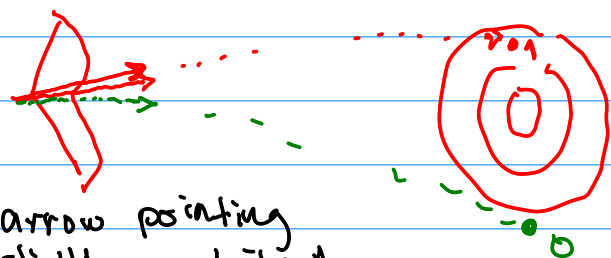
t	H(t)
2.1	1
1.9	1
2.01	1

$$H(t) = \begin{cases} 0 & \text{if } t<0 \\ 1 & \text{if } t\geq 0 \end{cases}$$



$\lim\limits_{t\to 0} H(t)$ = does not exist
DNE

t	$H(t)$
.1	1
.01	1
−.1	0
−.01	0
−.0001	0
+.0001	1



arrow pointing slightly up hits 1

arrow pointing slightly down hits 0

No approximating any identifiable result as $t\to 0$.

On the other hand if $t>0$ then $H(t)=1$ so

$$\lim_{t\to 0^+} H(t) = \lim_{\substack{t\to 0\\ t>0}} H(t) = \lim_{\substack{t\to 0\\ t>0}} 1 = 1$$

from the right

On the other hand if $t<0$ then $H(t)=0$ so

$$\lim_{t\to 0^-} H(t) = \lim_{\substack{t\to 0\\ t<0}} H(t) = \lim_{\substack{t\to 0\\ t<0}} 0 = 0$$

from the left

2 Definition of One-Sided Limits We write

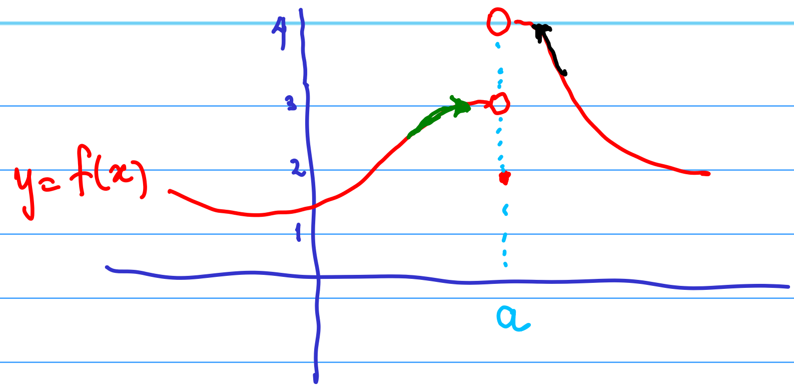
$$\lim_{x \to a^-} f(x) = L$$

and say the **left-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the left**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x *less than* a.

2 Definition of One-Sided Limits We write

$$\lim_{x \to a^+} f(x) = L$$

and say the **~~left~~ right-hand limit of $f(x)$ as x approaches a** [or the **limit of $f(x)$ as x approaches a from the ~~left~~ right**] is equal to L if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a with x ~~*less*~~ greater *than* a.



$$\lim_{x\to a^+} f(x) = 4$$

$$\lim_{x\to a^-} f(x) = 3$$

4 Intuitive Definition of an Infinite Limit Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

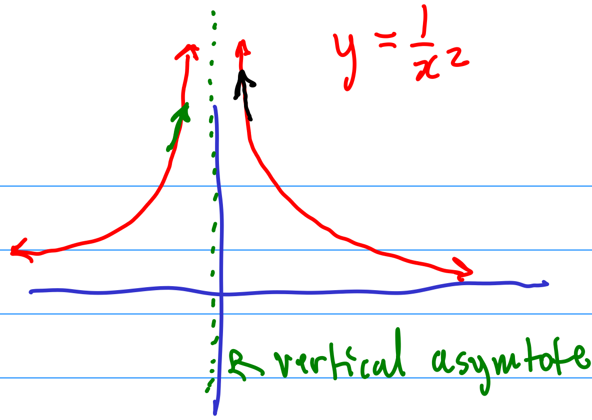
better and better approx. of ∞

5 Definition Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

better and better approx of $-\infty$



$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$

$\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\lim_{x \to 0} \frac{1}{x^2} = \infty$

If the limit from left is = from right then general limit is the same

The Quiz on Thursday covers

- Homework 1: Section 2.2#4,5,6,7,8,9

Note this homework is not to turn in, only to prepare for the quiz and aid in understanding.

Example from after class

4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

a. $\lim_{x \to 2^-} f(x) = 3$

b. $\lim_{x \to 2^+} f(x) = 1$

c. $\lim_{x \to 2} f(x)$ = does not exist because (a) and (b) are different.

not calculus → d. $f(2) = 3$

e. $\lim_{x \to 4} f(x) = 4$

not calculus no point on graph → f. $f(4)$ = DNE