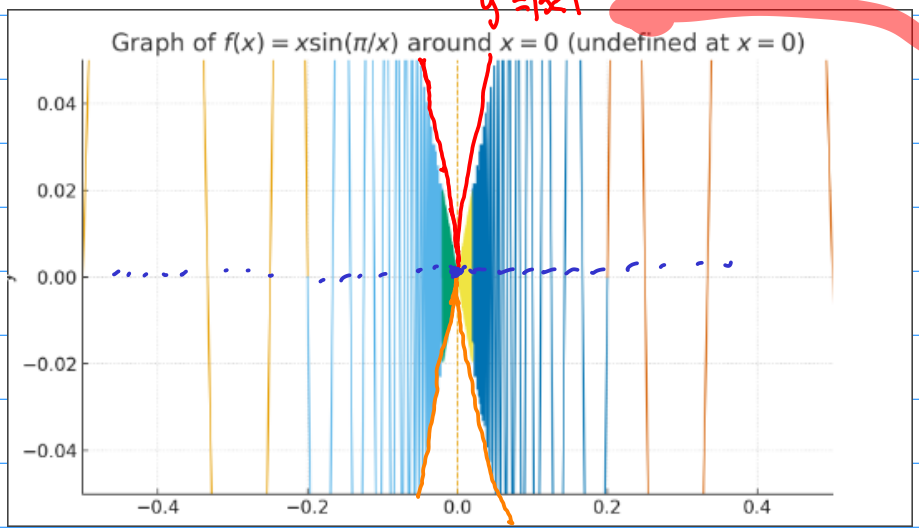

$$-|x| = \begin{cases} -x & \text{for } x>0 \\ x & \text{for } x<0 \end{cases} \leq g(x) \leq \begin{cases} x & \text{for } x>0 \\ -x & \text{for } x<0 \end{cases} = |x|$$

11. $\lim_{x\to 5} \dfrac{x^2 - 6x + 5}{x - 5}$

12. $\lim_{x\to -3} \dfrac{x^2 + 3x}{x^2 - x - 12}$

13. $\lim_{x\to 5} \dfrac{x^2 - 5x + 6}{x - 5}$

14. $\lim_{x\to 4} \dfrac{x^2 + 3x}{x^2 - x - 12}$

21. $\lim_{h\to 0} \dfrac{\sqrt{9 + h} - 3}{h}$

22. $\lim_{u\to 2} \dfrac{\sqrt{4u + 1} - 3}{u - 2}$

23. $\lim_{x\to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

24. $\lim_{h\to 0} \dfrac{(3 + h)^{-1} - 3^{-1}}{h}$

Try 14...

14. $\lim_{x\to 4} \dfrac{x^2 + 3x}{x^2 - x - 12}$

① Simplify

② Apply limit laws and continuity.

What if I skip step ① . . .

$$\lim_{x\to 4}\frac{x^2+3x}{x^2-x-12} = \frac{\lim_{x\to 4} x^2+3x}{\lim_{x\to 4} x^2-x-12} = \frac{16+12}{16-4-12} = \frac{28}{0}$$

OH NO!

Simplify $(x-4)(x+3)$

know this factor since plugging in 4 gave 0,

$$\frac{(x+3)x}{(x-4)(x+3)} = \frac{x}{x-4}$$

Now apply limit laws:

$$\lim_{x\to 4}\frac{x}{x-4} = \frac{\lim_{x\to 0} x}{\lim_{x\to 4} x-4} = \frac{4}{0}$$

$\neq$ NO!

Whenever I see something non-zero divided by zero this is the situation of a vertical asymptote.

Choices either limit is $+\infty$, $-\infty$ or doesn't exist.

$\frac{x}{x-4}$ what happens when $x\to 4$?

Case $x>4$ then $x-4>0$ and $\frac{x}{x-4}$ is large positive

$$\lim_{x\to 4^+}\frac{x}{x-4} = \infty$$

Case $x<0$ then $x-4<0$ and $\frac{x}{x-4}$ is very negative

$$\lim_{x\to 4^-}\frac{x}{x-4} = -\infty$$

Therefore left and right limits are different, so

$$\lim_{x \to 4} \frac{x}{x-4} = \text{does not exist}$$

so

$$\lim_{x \to 4} \frac{x^2 + 3x}{x^2 - x - 12} = \text{does not exist}.$$

Back to simplify step...

$$\frac{x^2 + 3x}{x^2 - x - 12}$$

I factored and cancelled

$$\begin{array}{r} 1 \\ x^2-x-12 \overline{)\, x^2+3x} \\ \underline{x^2-x-12} \\ 4x+12 \end{array}$$

$$\frac{x^2 + 3x}{x^2 - x - 12} = 1 + \frac{4x+12}{x^2-x-12}$$

① Simply
② Use limit laws

+1

23. $\lim_{x \to 3} \dfrac{\frac{1}{x} - \frac{1}{3}}{x - 3}$

Simplify (common denominator) +1

$$\frac{\frac{1}{x} - \frac{1}{3}}{x-3} = \frac{1}{x-3}\left(\frac{1}{x}\cdot\frac{3}{3} - \frac{1}{3}\cdot\frac{x}{x}\right) = \frac{3-x}{(x-3)3x} = \frac{-1}{3x}$$

(-1)

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3} = \lim_{x \to 3} \frac{-1}{3x} = \frac{-1}{3\cdot 3} = \frac{-1}{9}$$ ☑

5/5

4/5

Back to the first problem

$$\lim_{x \to 4} \frac{x^2 + 3x}{x^2 - x - 12}$$

2/5 or 3/5

① Simplify

can't simply like this

$$\frac{x^2 + 3x}{x^2 - x - 12} = \frac{3x}{-x-12}$$

try $x=1$

$$\frac{1+3}{1-1-12} = \frac{4}{-12}$$

$$\frac{3}{-1-12} = \frac{3}{-13}$$

② use limit laws

$$\lim_{x \to 4} \frac{3x}{-x-12} = \frac{\lim_{x \to 4} 3x}{\lim_{x \to 4} -x-12} \qquad \frac{12}{-4-12} = \frac{12}{-16} = \frac{6}{-8} = \frac{3}{-4}$$

Chapter 2.4

2 Precise Definition of a Limit Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - a| < \delta \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$