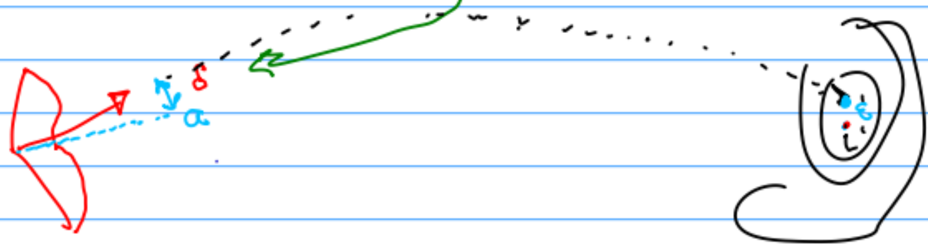
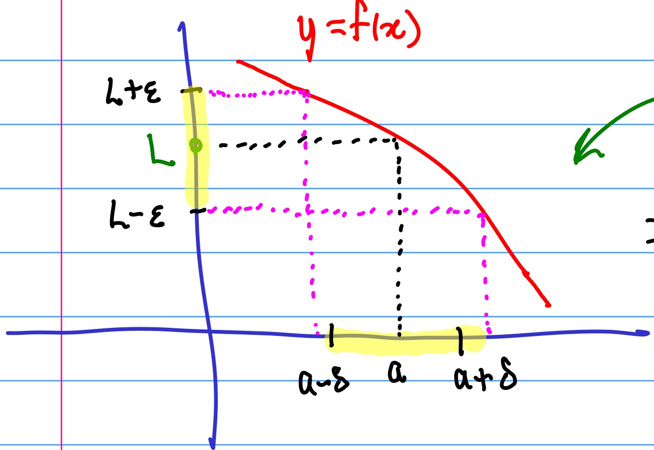
$\boxed{2}$ **Precise Definition of a Limit** Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |x - a| < \delta \qquad \text{then} \qquad |f(x) - L| < \varepsilon$$



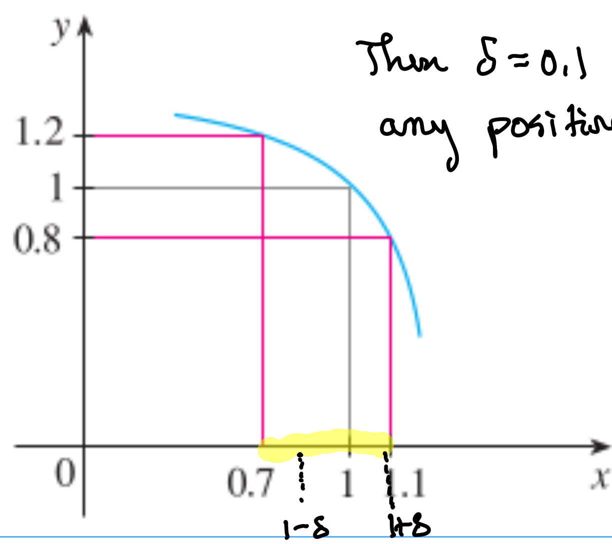


This graph can be described the the inequalities

If $0 < |x-a| < \delta$ then $|f(x)-L| < \varepsilon$.

1. Use the given graph of f to find a number δ such that

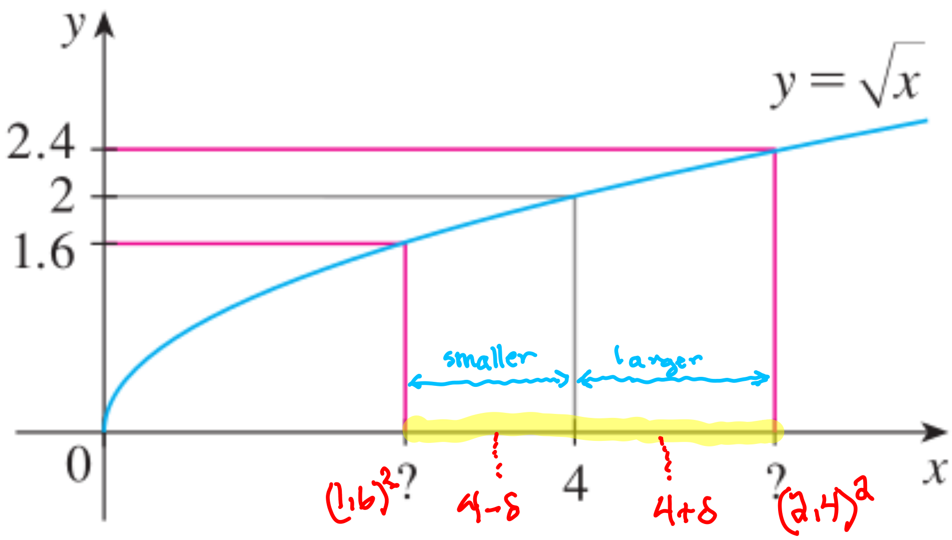
$$\text{if} \quad |x - 1| < \delta \qquad \text{then} \qquad |f(x) - 1| < 0.2$$



Then $\delta = 0.1$ will work as well as any positive value of δ less than that

3. Use the given graph of $f(x) = \sqrt{x}$ to find a number δ such that

$$\text{if} \qquad |x - 4| < \delta \qquad \text{then} \qquad |\sqrt{x} - 2| < 0.4$$

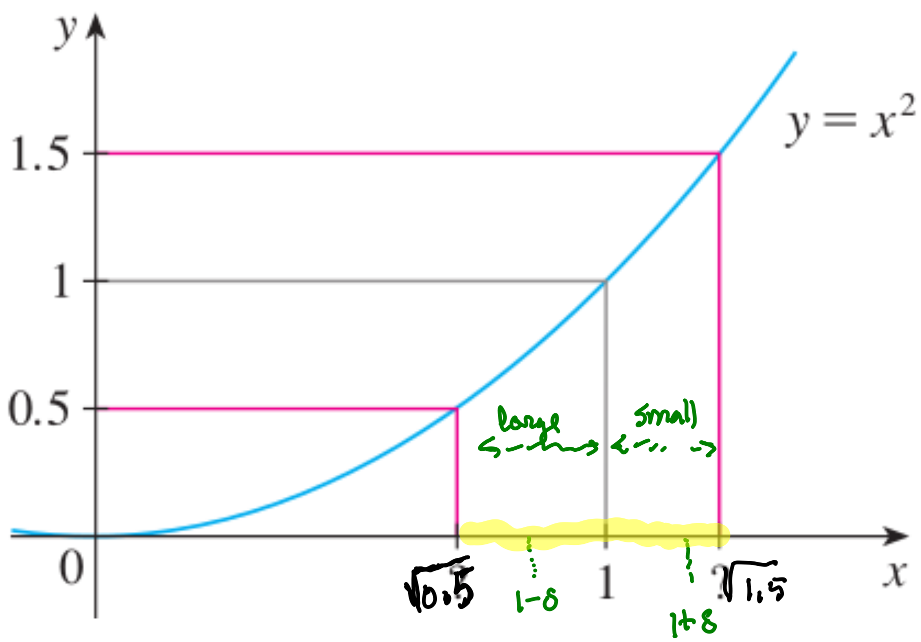


Then $\delta \approx \underbrace{4-(1.6)^2}_{1.44}$ or any positive value smaller works.

$$\begin{array}{r} 16 \\ \times\ 16 \\ \hline 96 \\ 16 \\ \hline 2.56 \end{array} \qquad \begin{array}{r} \overset{3}{\cancel{4}}.\overset{9}{\ }\overset{10}{\ } \\ -2.56 \\ \hline 1.44 \end{array}$$

4. Use the given graph of $f(x) = x^2$ to find a number δ such that

$$\text{if} \quad |x - 1| < \delta \quad \text{then} \quad |x^2 - 1| < \tfrac{1}{2}$$



Then $\delta = \sqrt{1.5} - 1$ or any positive value less works

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julia> sqrt(1.5)-1
0.22474487139158894
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372 ...

$\delta = 0.224$ works but rounding up to 0.225 is too big!

Prove the statement using the ε, δ definition of a limit.

20. $\lim_{x \to 10} \left(3 - \frac{4}{5}x\right) = -5$

solve back for this

Let $\varepsilon > 0$ be arbitrary. Choose $\delta = \boxed{5\varepsilon/4}$

Then $0 < |x - 10| < \delta$ implies

$$\left|3 - \frac{4}{5}x - (-5)\right| < \left|8 - \frac{4}{5}x\right| = \frac{4}{5}\left|8 \cdot \frac{5}{4} - x\right| = \frac{4}{5}|10 - x| < \frac{4}{5}\delta = \varepsilon$$

29. $\lim_{x\to 2}(x^2 - 4x + 5) = 1$

how close to the bullseye. aim.

Let $\varepsilon > 0$ be arbitrary. Choose $\delta = \boxed{\sqrt{\varepsilon}}$.

Then $0 < |x-2| < \delta$ implies $|x-2|^2 < \delta^2$

substitute somehow ...

Therefore

$$|x^2-4x+5-1| < |x^2-4x+4| = |(x-2)^2| = |x-2|^2 < \delta^2 = (\sqrt{\varepsilon})^2 = \varepsilon$$

almost 32

$$\lim_{x\to 3} x^3 = 27$$

Let $\varepsilon > 0$ be arbitrary. Choose $\delta = \min\left(1, \frac{\varepsilon}{4^2+3\cdot 4+9}\right)$

Then $0 < |x-3| < \delta$ implies $-\delta < x-3 < \delta$

so $3-\delta < x < \delta + 3$

$x < 4$

Therefore

$$|x^3-27| = |x-3||x^2+3x+9| < \delta|x^2+3x+9|$$

$$< \delta|(\delta+3)^2+3(\delta+3)+9|$$

$$< \delta|4^2+3\cdot 4+9| \le \varepsilon$$

$$\begin{array}{r} x^2+3x+9 \\ x-3\ \overline{)\,x^3-27} \\ -(x^3-3x^2) \\ \hline 3x^2-27 \\ -(3x^2-9x) \\ \hline 9x-27 \\ -(9x-27) \\ \hline 0 \end{array}$$